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Bundesamt für Meteorologie und Klimatologie MeteoSchweiz

Arbeitsbericht MeteoSchweiz Nr. 220

Optimization of Warning Systems based on Economic Criteria

Jacques Ambühl

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Ambühl, J: 2008, Optimization of Warning Systems based on Economic Criteria, *Arbeitsberichte der MeteoSchweiz*, **220**, 79 pp.

Herausgeber
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Contents

1 Introduction

A theoretical analysis of the relationship between a warning organisation and the users of the warnings issued by this organisation is presented in this report. The warning organisation is usually a weather service. Users might be farmers, hydrological services, airport authorities or management boards of power plants. The warning organisation will thereafter be referred to as the issuer, the user of the warnings as the addressee. They will be referred to as "the actors" when considered together. The methodology is inspired by numerous mathematical developments that occurred during the last decades in finance engineering, aimed at providing investors with optimal decisional schemes.

The programme is realised by establishing a clear distinction between the actors. The addressee is characterised by a dual risk and economic profile. His risk profile is composed on one side by the risks his business is confronted with (e.g. lost of a harvest in case of extreme precipitation), on the other side the climatology he is exposed to (e.g. the probability of occurrence of extreme precipitation). Both risk and climatology are then merged in the relative operation characteristics of the addressee.

His economic profile is inspired by the classical work of D. S. Richardson [Reference 1]. It is defined by the triad of the loss resulting in an unpredictable event for which neither mitigating nor protective actions were taken, the cost induced by the protective actions taken in case of occurrence as well as in case of non occurrence of an event, and finally the residual cost occurring in the case of a well predicted event for which protective measures were (adequately) taken. These various risks and economic factors being fairly entangled, an objective of this work is to provide, at least at theoretical level, some clarity in that matter.

The issuer is expected to base his warning on probabilistic forecasts emanating from an Ensemble Prediction System (EPS), or on any diagnostic system providing an answer expressed in term of probabilities. The performance of the issuer is then characterised by a dedicated Relative Operation Characteristic (ROC).

The maximization of the economic benefit of the addressee is realised through the adequate tuning of the warning system. The determination of optimal warning thresholds deserves this purpose. The dual approach proposed provides a quantitative relationship between the performance objectives having to be reached by the issuer and the monetary outcome expected by the addressee. This duality, enabling the assessment of the impact of the issuer's performance on the addressee's efficiency, is in full accordance with the requirements formulated in the realm of New Public Management projects undertaken in several weather services. It enables the settlement of a service level agreement between both actors.

Furthermore, an accurate knowledge of the profile of the addressee enables the elaboration of specifically customer-tailored products, in the same vein as structured products modified the financial realm decades ago.

2 Outline of the problem

At this point, it is worth sketching the differences between a warning system, an insurance contract and a weather derivative product. Weather derivatives are conditional contracts established between a finance institute and a stakeholder whose business is weather dependent. Weather derivative payouts solely depend on the outcome of the weather, regardless of how it affects the profit of the holder. On the contrary, the holder of an insurance contract has to prove that he has suffered a financial loss induced by an adverse weather event that actually occurred in order to be compensated. Insurance contracts are conceived in order to cover damages on real estates and objects, weather derivative to protect financial flows. Warning systems are aimed at guiding stakeholders, i.e. addressees, in the triggering of protective or mitigating actions in the prospect of adverse weather conditions.

In a first approximation, one can consider warning systems and insurance contracts as being designed to cope with extreme weather events. On the contrary, weather derivative are conceived in order to cover financial risks induced by rather normal weather fluctuations [Reference 2]. The boundaries between these businesses being fuzzy, the issue will be briefly addressed.

Figure 1 sketches a schematic relationship between these various domains.

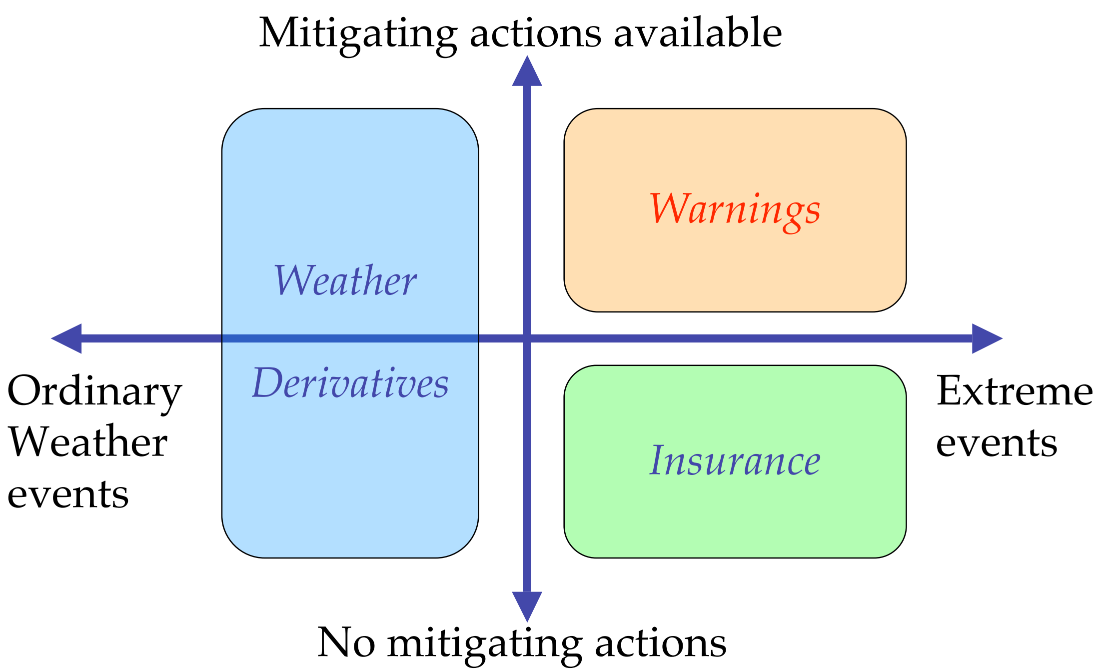

Figure 1: *Diagrammatic representation of the four possible combinations of common or extreme weather events, for which mitigating actions may or may not be available. Sketch of three corresponding business fields.*

3 Scope and Aim

This essay is devoted to the study of warning systems tuned in order to cope with extreme events, as sketched on the upper right corner of Figure 1. The approach is dualistic. It consists in establishing and adamantly maintaining a clear demarcation line between both actors, addressee and issuer, and then in analysing the interaction occurring between them.

The methodology is based on theoretical considerations whose consequences are systematically verified against numerical simulations.

The document is organised in two layers. Leading concepts and related discussions are introduced in the body of the text, Sections 4 to 11. Ancillary developments are provided in the Appendix, Section 12. The self-consistency of the core is as far as possible warranted, thus making references to the Appendix optional.

The aim of the project consists in establishing a conceptual model of a warning system tuned in order to operate optimally under prevailing climatic conditions and along the line of requirements defined by end users.

Acknowledgment

This study summarises the result of personal reflections conducted on private basis in 2006 and 2007. Martina Amstutz, Tamara Comment, Thomas Egli, Mathias Rotach, Philippe Steiner and Christophe Voisard deserve special thanks for their generous help.

4 Brief historic and basic definitions

During the last decades, various forecasting scores have been introduced in order to assess the quality and the efficiency of weather forecasting and warning systems. The widespread of probabilistic forecasts based on Ensemble Prediction Systems has favoured the diffusion of performance indicators based on such systems [Reference 3]. In this context, the probability of detection, the false alarm rate and false alarm ratio have emerged as the key parameters used for this purpose. In 2003, ECMWF published Recommendations on the verification of local weather forecasts, [Reference 4], having almost mandatory status among ECMWF member states, and emphasising the use of the aforementioned scores. Those terms are succinctly defined below.

The hit rate, expresses the ratio between the number of correctly warned events to the total number of events. Measuring the overall success of the warning system, it is of paramount importance for both actors.

On the other side, the difference between the false alarm rate and the false alarm ratio is subtle and should not be underestimated. The false alarm rate measures the frequency at which the addressee's business, instead of running smoothly, is unnecessarily impeded by protective actions taken under fair weather conditions. The false alarm ratio, expressed in terms of the number of mistakenly issued warnings divided by the total number of issued warnings, provides a measure of the quality of the service provided by the issuer. Although the hit rate is relevant to both actors, the false alarm rate is a measure of the efficiency of a warning system, as perceived by the addressee, and the false alarm ratio a measure of the performance of that warning system, in issuer's hand. This fact is illustrated through a brief numerical example presented in the Appendix, Section 12.1.

Conclusively, the false alarm rate appears to be an addressee's issue, the false alarm ratio an issuer's concern. The dual analysis of both profiles presented hereafter a direct consequence of that observation.

5 Addressee's profile

The object of this Section consists in elaborating the profile of a so-called rational addressee. It is based first on the careful accounting of weather related hazards he is facing to, then on the evaluation of the financial burden thereby induced. Rather than trying to compute a solid monetary outcome having to be paid, this Section is aimed at identifying and explaining the relationship between the statistical - climatological information available and the financial constraints the addressee is confronted with.

5.1 Addressee's risk profile

5.1.1 Probabilistic formulation of the risk

Instead of working with *a posteriori* computed frequencies, it is worthwhile to consider probabilistic functions representing the expected distributions of the events to be taken into consideration. Two of them are introduced in the present setting: on the one hand a representation of the climatology the addressee is exposed to, on the other hand the frequency of the disasters he is confronted with. Both are expressed in terms of one weather parameter, e.g. temperature or gale intensity or cumulated precipitation fallen within a period of time. This period of time, which can last a few hours, a day, a week, will be referred to as Δ.

The frequency of the disasters might be expressed as for example in terms of the relative increase of medical emergencies occurring during a heat wave or in terms of the frequency of interventions of emergency crews in the case of a storm, all of them being expected to occur during Δ.
The cost induced by one disastrous event is L, expressed in monetary units. The span of the meteorological parameter has to be specified as well, for example between 0 and 200 km/h for gales (at least in Switzerland), or between 0 and 400 mm precipitation within Δ. Those inferior and superior bounds are referred as *0* and B in the following. The meteorological parameter will simply be expressed in arbitrary ***units***. More specifically, the two following probabilistic distributions are now introduced:

- Climatic profile, Figure 2, probability of occurrence, during the period Δ, of a weather event W, the intensity of which lies between q and $q + dq$:
$$C_{(q)}dq = \Pr[W \in [q, q + dq]\ ;\ \Delta]\ ;\ q \in [0, B] \qquad (1)$$

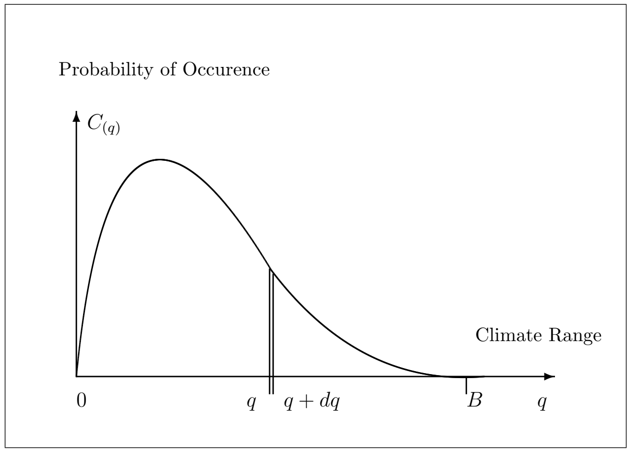

Figure 2: *Climatic profile: Probabilistic distribution of weather events.*

- Disaster profile, Figure 3: probability of occurrence during the period Δ of a disaster D induced by a weather event W the intensity of which lies between q and $q + dq$:

$$E_{(q)}dq = \Pr[D \in \Delta;\, W \in [q, q + dq]];\; q \in [0, B] \tag{2}$$

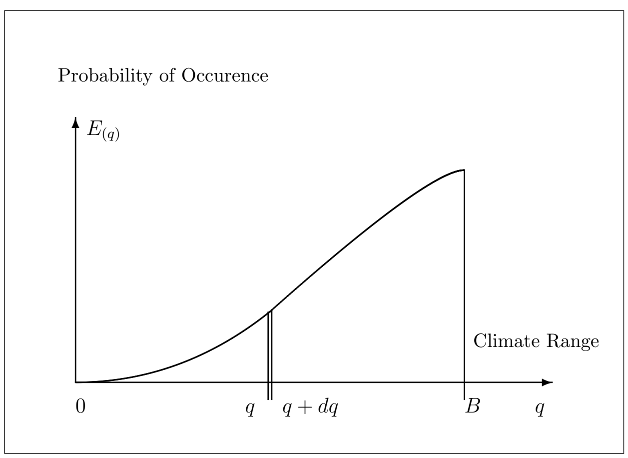

Figure 3: *disaster profile: Probabilistic distribution of induced disasters*

Both distributions are presented again in Figure 4, this time with a ***meteorological threshold*** Q sketched as the downward pointing vertical arrow. One notices in that figure that most meteorological events occur at low intensity and are figured by the belly of the C distribution. Extreme events are represented by the right tail of the C distribution. Accordingly, as the probability of induced disasters grows for increasingly extreme weather events (E distribution), the addressee is confronted with the dilemma of having to determine an optimal meteorological threshold Q at and beyond which mitigating actions should be undertaken. The introduction of hit and false alarm rates, as well as the corresponding relative operation characteristic, will deserve this purpose.

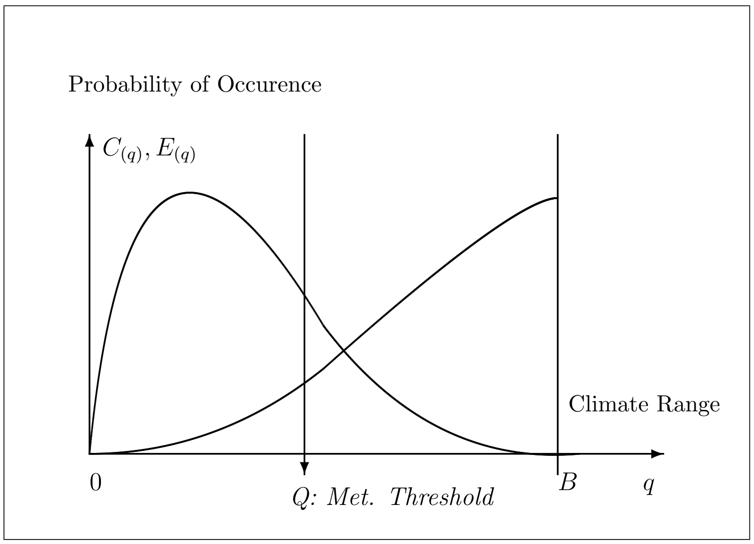

Figure 4: *Climatic versus disaster profiles: Basis for the construction of the relative operation characteristic.*

5.1.2 Relative Operation Characteristic

Hit rate and false alarm rate are presented in the Appendix, Section 12.1, as arithmetic ratios between accumulated cases. Using the definitions introduced in the previous Section and following the development given in the Appendix, Section 12.2, it is possible to construct the hit rate and the false alarm rate as functions depending on the probabilistic distributions of climatic and disaster profiles. They are defined as:

The Hit Rate: expresses the ratio between the probability of disasters occurring in the domain $[Q, B]$, for which mitigating actions are undertaken and the overall probability of occurrence of weather induced disasters:

$$Hr_{(Q)} = \frac{1}{\Omega} \int_Q^B E(q)C(q)dq \tag{3}$$

The False Alarm Rate: expresses the ratio between the probability of non-occurrence of disasters in the domain $[Q, B]$ for which mitigating actions are inadequately undertaken and the overall probability of occurrence of weather conditions not triggering disasters:

$$Far_{(Q)} = \frac{1}{1-\Omega} \int_Q^B (1 - E(q))C(q)dq \tag{4}$$

These expressions, taking their values in the interval $[0, 1]$, are functions of the meteorological threshold Q. In both of them, $\Omega = \int_0^B E(q)C(q)dq$, measures the overall probability of occurrence of weather induced disasters.

Having the hit rate and false alarm rate at hand, the relative operation characteristic of the addressee can be established. Presented in Figure 5, it is given by the curve whose abscissa is expressed in terms of false alarm rate, whose ordinate is expressed in terms of hit rate and which is parametrized in terms of meteorological thresholds Q. The parameter runs from low meteorological thresholds (corresponding to high hit rates and false alarm rates), to be found at the top right corner of the diagram, to high meteorological thresholds (accordingly corresponding to low hit rates and false alarm rates) at its bottom left corner. In between, the ROC-curve moves into the vicinity of the top left corner, where the hit rate is rather high, the false alarm rate rather low.

A sound addressee would evidently choose a threshold in this area. Indeed, addresses who choose low thresholds are ***risk adverse***. They are ready to trigger mitigating actions even by faint evidence of an incoming disaster. Consequently, they pay with a substantial increment in false alarm rate a marginal improvement in hit rate. Correspondingly, those addressees who decide to operate at high thresholds (with low false alarm rate and hit rate) are described as ***risk friendly***. They are willing to take mitigating actions only when the evidence for an incoming disaster is high. They also require a substantial increase in hit rate in order to endorse even a tiny increment in false alarm rate.

It is worth noticing that the kind of awareness presented here focuses on the impactive risk on the addressee's business. In a dual interpretation, the issuer's point of view might be considered. In that dual perspective, an issuer giving out warnings at low thresholds, thus possibly warning too frequently, would happen to behave in a risk friendly manner. Risk awareness will be further considered and refined in Sections 5.3 and 6.4.

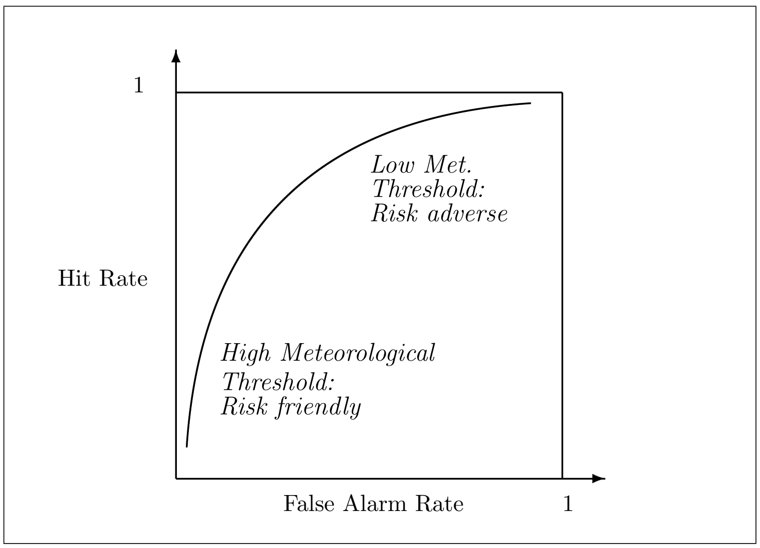

Figure 5: *Addressee's risk awareness interpreted on the relative operation characteristic*

Enabling the fusion of the information related to the occurrence of climatic events and their consequences onto the addressee's business, the relative operation characteristic is being considered in the following as the ***risk profile of the addressee***. Accordingly, the meteorological threshold is the governing parameter through which the addressee controls the amount of risk he is willing to cope with (Figure 5). A methodology has therefore to be introduced in order to enable the computation of an optimal meteorological threshold for the addressee. Considerations related to his economic profile will enable us to resolve this indeterminacy.

The concepts discussed so far are presented in a numerical simulation given in Figure 6. An example of the relationship between climatic and disaster profiles with a meteorological threshold arbitrarily set at $Q = 7.5$ units is presented on the top panel. In this set-up, the addressee experiences no damage for events occurring at an intensity smaller than or equal to 5 units. The corresponding relative operation characteristic is given on the bottom panel. The meteorological threshold is marked by the green dot. By very low thresholds, below 5 units, mitigating actions being always taken, the hit rate equals 1 and the false alarm ratio is high. Both converge toward zero by high thresholds.

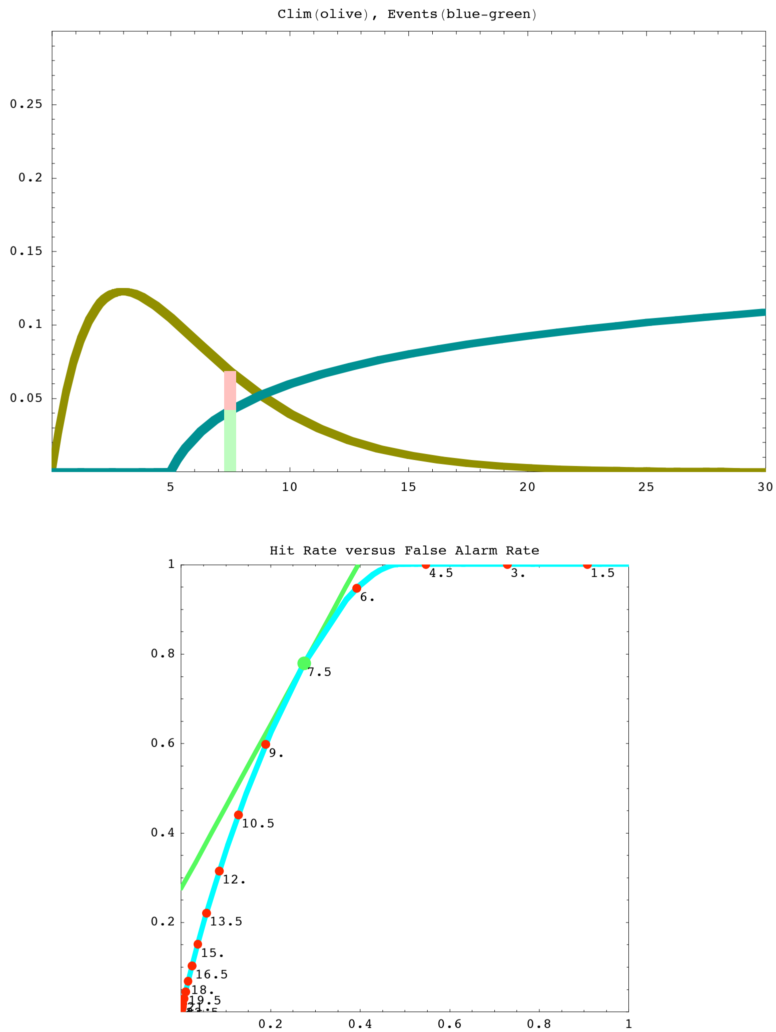

Figure 6: *Top panel: example of the relationship between climatic and disaster profiles with a meteorological threshold arbitrarily set at* $Q = 7.5$ *units. Abscissa: climate range; ordinate: probability of occurrence. Bottom panel: corresponding relative operation characteristic. Abscissa: false alarm rate; ordinate: hit rate. The straight line tangent to the disaster profile at 7.5 units is related to the addressee's economic profile that will be introduced in following Section.*

5.2 Addressee's economic profile

The profile is expressed following the definitions and a slight generalisation of the theory proposed by Richardson [Reference 1]. This theory provides a measure of the economic impact induced by the occurrence of four possible situations described in the following contingency Table 1. The elements are expressed in monetary units: L represents the loss induced by a disaster for which neither a warning was issued nor mitigating actions were taken. C represents the costs induced by mitigating actions. They are due in case of occurrence of a correctly warned event, as well as in case of a mistakenly warned non event. λ represents residual disaster costs remaining in the case of a correctly warned event. It is assumed that $\lambda \ll L$ and $C < L$.

	no event	event
mitigating actions undertaken	C	$C + \lambda$
no mitigating actions undertaken	0	L

Table 1.

Although not present in the original paper by Richardson, the λ parameter is easily introduced and happens to provide a valuable generalisation.

The average costs M the addressee is faced to during a period long enough to be of climatological relevance can now be evaluated. They are given by:

$$M = \frac{1}{a+b+c+d}\left[bL + Cc + (C+\lambda)d\right] \tag{5}$$

with a, b, c and d defined in Appendix, Section 12.1. Following Richardson, M can be expressed in terms of frequency of occurrence of the event (Ω), hit rate (Hr) and false alarm rate (Far):

$$M_R(Hr, Far) = L\left[Far\,\Gamma\,(1-\Omega) + Hr\,\Omega\,(\Gamma + \Lambda - 1) + \Omega\right] \tag{6}$$

The derivation of this expession is provided in Appendix, Section 12.5. Besides the ***cost-loss ratio*** $\Gamma = \frac{C}{L}$, the pivotal parameter in this study, the parameter $\Lambda = \frac{\lambda}{L}$, baptized ***residual-loss ratio***, is introduced as well. Expression (6), whose Graph is presented in Figure 7, is the exact equivalent of equation (5), however expressed in terms of hit rate and false alarm rate.

The Richardson's economic function, being expressed in hit rate and false alarm ratio, can be projected onto the ROC-frame $\{[0,1] \times [0,1]\}$. Being linear in hit rate as well as in false alarm ratio, its isolines, computed for a constant monetary values $M_\$$ by $M_R(Hr, Far) = M_\$$, are straight lines in the ROC-frame. Sketched in Figure 8, they are called ***iso-costs*** in the following: as $[Hr, Far]$ moves along an iso-cost, no change occurs to the burden the addressee is faced to. Indeed, from his perspective, iso-costs are to be understood as lines of equal sensitivity that might accordingly have been baptised iso-tolerance lines.

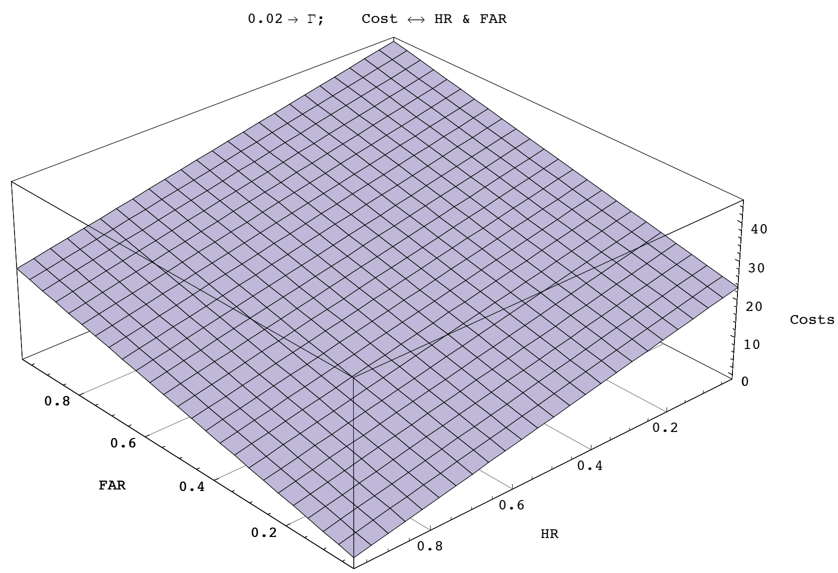

Figure 7: *Richardson's economic function. The hit rate runs along the right edge of the cube, the false alarm rate along the left edge. The cost-loss ratio is arbitrarily fixed at* Γ *= 0.02. The function being linear, its graph is a plane in the space [false alarm rate* $\times$ *hit rate* $\times$ *Costs].*

A simple analysis of the structure of the Richardson expression (6) shows that the slope of the iso-costs, expressed as $\frac{\Delta Hr}{\Delta Far}$, is governed by the value of the cost-loss ratio Γ (Figure 8). Thus, expressed in the ROC-frame, the iso-costs and their slopes represent the synthetic parametrization of the economic profile of the addressee, with the cost-loss ratio acting as the governing parameter.

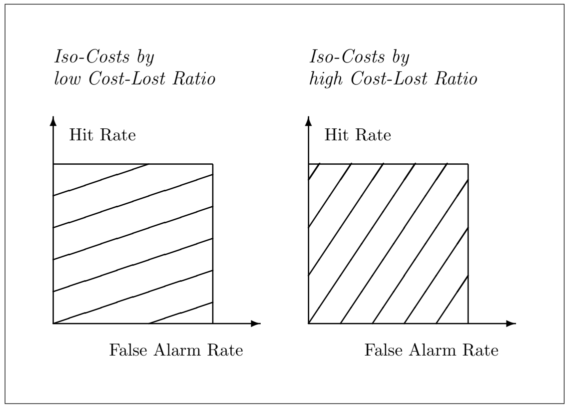

Figure 8: *Slope of the iso-costs in relation to cost-loss ratios. Costs are minimum at the top left corner of the diagram, maximum at the bottom right corner. iso-costs are interpreted as the addressee's economic profile in the following.*

Addressees do indeed frequently ignore the actual value of their own cost-loss ratio. Few examples [given in Reference 3] and one documented statistic for heat wave induced mortality in Switzerland, [Reference 5], are given here for illustration (with Λ parameter set to zero):

- $\Gamma = 0.02 - 0.05$ for orchardists (Murphy 1977)
- $\Gamma = 0.01 - 0.12$ for fuel-loading of aircraft (Leigh 1995)
- $\Gamma = 0.125$ for winter road gritting (Thornes and Stephensen 2001)
- $\Gamma = 0.03 - 0.18$, Mortality and temperature in Switzerland 1990 - 2003 (Herren, personal communication and Reference 5)

It should be deemed cynical to interpret mortality in terms of cost-loss ratio. Indeed, the relationship between the latter and the disaster profile will be demonstrated below in Section 5.3.

Up to now we have seen that the meteorological threshold is the parameter through which the addressee controls the amount of risk he is willing to cope with. It appears here that the cost-loss ratio plays a comparable role, this time emanating from the economic perspective. Although the addressee's freedom is likely to become restricted by the interplay between those two governing parameters, this interplay will contribute to clarify the risk versus reward dilemma entangled in the issue and enable us to define the profile of a rational addressee.

5.3 Rational addressee: cost-loss ratio expressed in terms of meteorological threshold

The key elements having to be taken into consideration by the addressee when determining an optimal meteorological threshold Q are now at our disposal. His risk profile, considered first, accomplishes the fusion of the information emanating from both climatic and disaster profiles, as defined by expressions (1) and (2). It is conveyed in terms of relative operation characteristic through the hit rate and the false alarm rate, equations (3) and (4). On the other side, the Richardson model of costs and losses, equation (6), describes his economic profile, at least for the issue at stake.

Both profiles being expressed in terms of hit rate and false alarm rate, they can be transported onto the ROC-frame, as presented in Figure 9 below for two exemplary values of the cost-loss ratio. Considering that the addressee will seek his minimal financial burden in the long term, he will choose on the ROC the meteorological threshold providing the minimum value of the economic Richardson's function.

Noticing that the iso-costs are straight and the ROC concave, it appears that the optimal meteorological threshold is naturally defined at the point of tangency of the iso-costs with the ROC (Figure 9). Thanks to the linearity of the iso-costs and the concavity of the ROC, it is that unique point of the ROC where the economic function reaches its minimum. According to the characterisation of risk awareness presented earlier, one immediately notices in Figure 9 that the addressee adopts a fairly risk adverse strategy when his cost-loss ratio is low, reciprocally a comparatively more risk friendly strategy when it is high.

As already mentioned, discussions with addresses make clear that they frequently ignore their cost-loss ratio, as well as the existence of a possible relationship with the meteorological threshold at which they might be warned. Indeed, they are faced with the following alternative:

The addressee knows his cost-loss ratio Γ: In this case, the slope of the iso-costs being given, seeking a minimum of his economic burden,

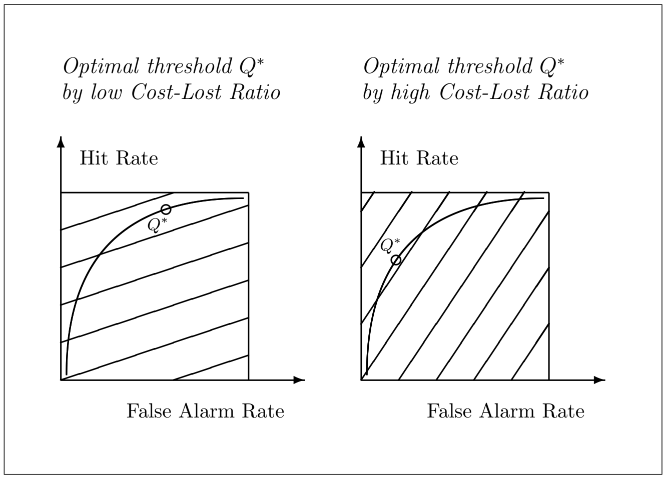

Figure 9: *Fusion of the risk profile (ROC) and the economic profile (iso-costs) of the addressee. The minimum of the addressee's financial burden is reached at the tangency point of the ROC with the iso-costs. This defines the value optimal meteorological threshold (Q^* - circles).*

he is bound to choose the meteorological threshold Q^* at that point on the ROC where tangency with the iso-costs occurs.

The addressee has fixed a meteorological threshold Q^*: Considering his choice as being economically optimal, he has implicitly assumed that the slope of the iso-costs are parallel to the tangent to the ROC at that chosen meteorological threshold. As a matter of fact, he has defined his cost-loss ratio Γ in the same fallen swoop.

Expecting a rational behaviour from the addressee, it is tempting to use the information available in order to formalise the issue. This information is threefold: it emanates 1) from the risk profile, provided by the ROC, 2) from the economic profile, as described by the iso-costs, and 3) from the requirement for optimality, expressed as the tangency requirement between iso-costs and ROC. According to the definitions established so far, this formal relationship is to be expressed in terms of meteorological threshold on the one side, in terms of cost-loss ratio on the other side. The connection between both sides is provided by the tangency requirement. It expresses the best addressee's trade-off between the variation of hit rate and the variation of false alarm rate. Those ratios having to be equal at the tangency point on the ROC, one has, as shown on Figure 9:

$$\frac{\partial_Q Hr}{\partial_Q Far}|_{Riskprofile} = \frac{\partial Hr}{\partial Far}|_{Economicprofile}$$

This geometrical requirement defines a simple partial differential equation whose solution, presented in Appendix, Section 12.3, provides the expected relationship between the cost-loss ratio and the meteorological threshold:

$$\Gamma_{(Q)} = \frac{C}{L} = (1 - \Lambda)\, E_{(Q)} \tag{7}$$

This expression is of startling simplicity. It expresses the proportionality between the disaster profile $E_{(Q)}$ at a threshold Q and the cost-loss ratio, and equality between both if the residual-loss ratio, given by Λ, is set to zero. It is worth noticing that the Ω factor, representing the climate component, vanishes in the course of the derivation. Conclusively,

The addressee is qualified as rational if the relationship between his meteorological threshold Q^* and his cost-loss ratio Γ expresses the subsequent geometrical construction (Figure 10) and therefore satisfies equation (7) :

The notions of risk awareness and rationality are independent. According to his economic profile, an addressee can be rational **and** risk adverse, as well as rational **and** risk friendly or even rational **and** risk neutral. In addition, it will be shown in Section 7.3 and in Appendix, Section 12.6, that economic aspects themselves vanish by the optimal tuning of the warning system, leaving only the disaster profile of the addressee and the forecasting skill of the issuer in balance.

An improved definition of the risk awareness can now be proposed: An addressee will be considered risk adverse if it appears that, for him, the ratio $\dfrac{\Delta Hr}{\Delta Far}$ is smaller than one, respectively risk friendly if this ratio is larger than one. Expressing the ratio of the variation of two dimensionless quantities, risk awareness is itself a purely differential concept given without monetary value of any kind. It must be furthermore emphasised that the requirement for $\dfrac{\Delta Hr}{\Delta Far} \approx 1$ does not define rationality, but merely expresses the notion of risk neutrality.

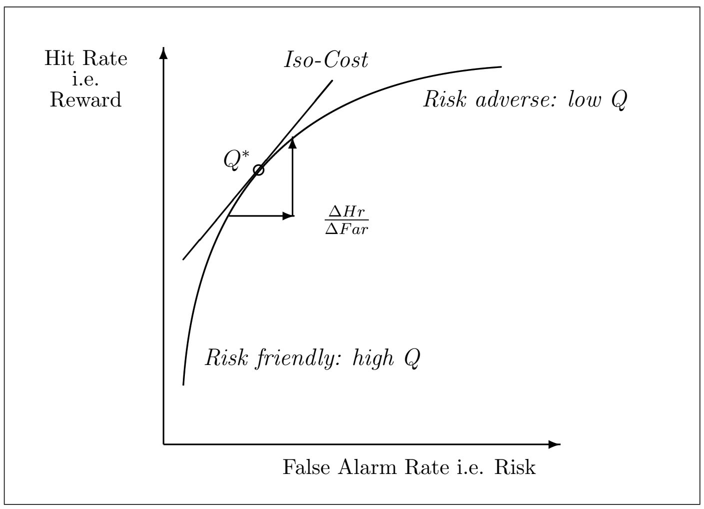

Figure 10: *Zoom on the tangency point between iso-cost and ROC for a* ***rational addressee****: being located at the point on the ROC where the Richardson function takes its minimum, the optimal meteorological threshold Q^* expresses the best possible $\frac{\Delta Hr}{\Delta Far}$ ratio for the addressee, according to his economic profile.*

Reference [6] provides an excellent introduction in the theory of risk awareness applied in financial mathematics.

Finally, notwithstanding the fact that sensible readers may feel some distaste towards such crude definitions of rationality and risk awareness, it should be remembered that the present work is aimed at building conceptual models whose least praised quality should not be their utmost simplicity.

5.4 Summarising example

As a conclusion of this Section and in order to exemplify the situation here and now, two addressees are presented in Figure 11. Both are characterised by the same risk profile, figured in the top row. Their economic profiles, however, differ (middle row). The left addressee, having a low cost-loss ratio, operates at low meteorological threshold (6 units). Conversely, the right addressee operates at higher cost-loss ratio and meteorological threshold (12 units). Whilst the former is fairly risk adverse and the latter definitely risk friendly, both behave rationally.

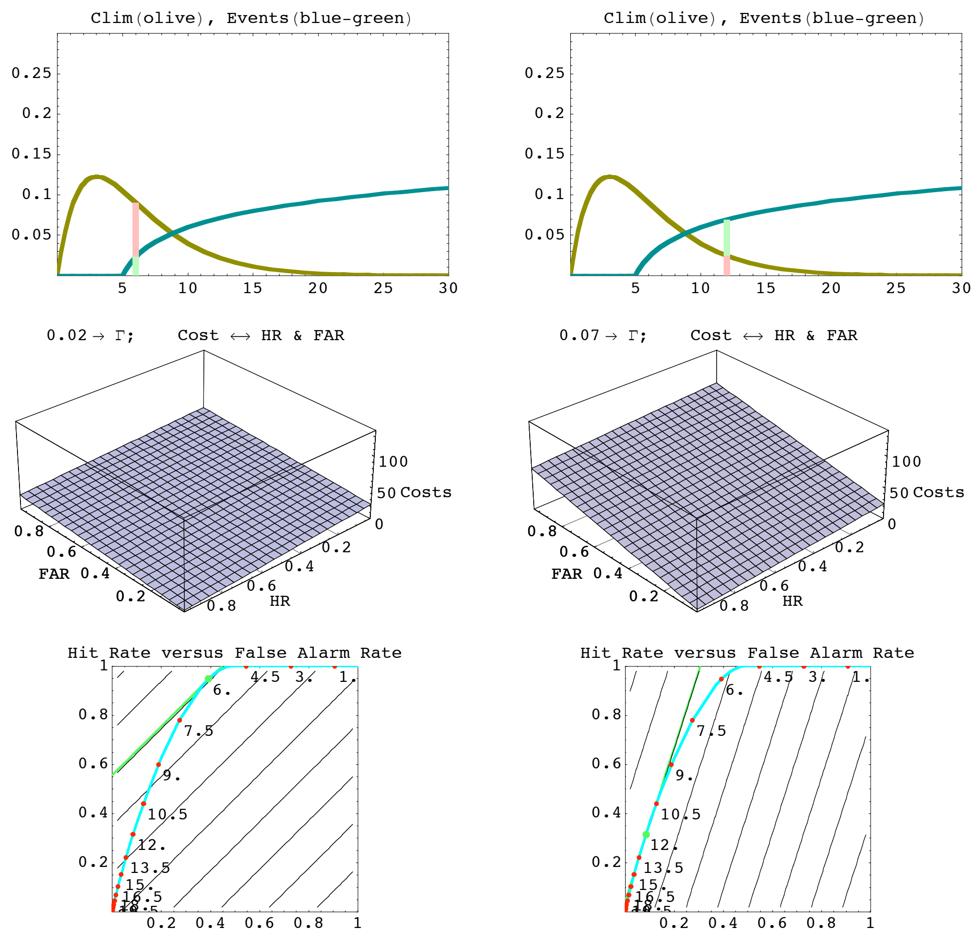

Figure 11: *Comprehensive representation of the relationship between meteorological thresholds and cost-loss ratios for two rational addressees. Left column: meteorological threshold = 6 units; cost-loss ratio = 0.02. Right column: meteorological threshold = 12 units; cost-loss ratio = 0.07. The addressee is fairly risk adverse on the left column, definitely risk friendly on the right one. This Figure is related to Figure 22 in Section 10.*

6 Issuer's profile

Having discussed the addressee's characteristics so far, we will now focus our attention on the issuer, with meteorological services in mind. Considering that meteorological warnings are increasingly based on probabilistic forecasts, possibly emanating from Ensemble Prediction Systems, such a system will be simulated first, thus enabling the extraction of the information relevant to the current project. Later, the addressee's profile described in the previous Section will be injected onto the issuer's performance chart, expressed as ROC of hit rate and false alarm ratio. The synthesis of both actor's perspectives will then be presented and discussed in Section 7.

6.1 Conceiving a warning system based on a simulated Ensemble Prediction System

A little bit of shrewdness is far from being unhelpful in designing the contraption which, although avoiding the overwhelming complexity of a real Ensemble Prediction System (EPS), produces pertinent simulations of probabilistic forecasts. The methodology applied here, consisting in building a sequence of random generators eventually producing the required probabilistic items, is presented in the Appendix, Section 12.4.

Characteristics of the simulated probabilistic forecasts and the corresponding weather events are presented in Figure 12. The climatic range in which simulated weather events occur is represented by the abscissa, spanning from 0 to B. The probability of their occurrence is expressed on the ordinate. The downward pointing thin arrow determines a meteorological threshold Q. A probabilistic forecast for the next verifying time is sketched as the (near Gaussian) distribution curve. The corresponding weather event of intensity $\tilde{Q}$ is represented as the downward pointing thick arrow. Weather events as well as their corresponding forecasts are simulated in accordance with the probabilistic distribution of weather events given in Figure 2. This ensures that the addressee and the simulated forecasting model are subjected to the same climatology, of course a prerequisite for the envisaged confrontation of both actor's profiles.

The area below the probability distribution located at the right of the meteorological threshold Q, given by $\int_Q^B P(q)dq$, expresses the probability of the event "Weather occurs with an intensity equal or greater than Q", as forecasted by the simulated Ensemble Prediction System. (It is assumed that $\int_0^B P(q)dq = 1$). Having these elements at hand, it suffices now to introduce besides the meteorological threshold Q a ***probability threshold*** P to define a simulated warning system.

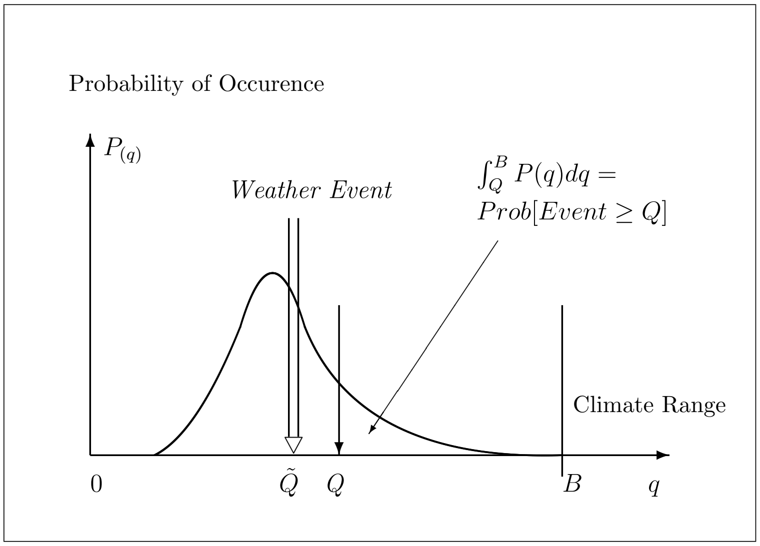

Figure 12: *Decision based on a probabilistic forecast. Q is the meteorological threshold. The area below the probability distribution located at the right of the meteorological threshold Q expresses the probability of the event "Weather occurs with an intensity equal or greater than Q", as forecasted by the Ensemble Prediction System.*

All elements required to design the issuer's contingency table, *defined for one meteorological threshold Q and one probability threshold P*, are now at our disposal. Based on a table $< event, forecast >$ produced by the EPS-simulator (Appendix, Section 12.4) and making use of the decision scheme given hereafter, they enable the construction of an arithmetic contingency table having the same structure as those tables presented in Appendix, Section 12.1. It must be stressed that the scheme proposed here belongs to a vast array of methods having been designed since the beginning of scientific meteorology in order to issue probabilistic forecasts. Three of them, deriving probabilistic short range forecasts from a deterministic high resolution model, or from radar information [References 7, 8 and 9] are mentioned here as possible examples.

A decision scheme based on probabilistic forecasts has now to be implemented. Elaborating such decision schemes, based upon ensemble or any other kind of probabilistic forecasts, is always an awkward business. Correspondingly, the scheme proposed in the next Section seeks simplicity.

6.2 Defining a decision scheme

Innumerable decision support systems have been conceived and implemented since time immemorial in all kinds of human affairs. Those used in environmental sciences, as for example in meteorology, have for a long time been based upon observation, experience and memory. Scientific analysis, quantified observation and numerical simulation shape modern meteorological decision schemes. However, even if human forecasters refer to those comprehensive technological systems when taking decisions, they cannot help letting personal bias playing a more or less concealed role in their cogitations.

The choice of a decision scheme is already a decision *per se*, indeed a "*meta-decision*". In this perspective, seeking rationality and aimed at inhibiting any potential psychological slant, the system is implemented as a simple automatic algorithm operating in accordance to the rules given in Table 2:

Threshold	Units	$\tilde{Q} < Q$	$\tilde{Q} \geq Q$
	Event	did not occur	did occur
Probability	Alarm		
$\int_Q^B P(q)dq \geq P$	issued	false alarm	successful alarm
$\int_Q^B P(q)dq < P$	not issued	correctly rejected	missed event

Table 2.

Reflecting on the interplay between the probability and the meteorological thresholds, the attentive reader will have noticed that, the latter being determined by the addressee, the former is likely to become the governing decisional parameter in the issuer's hands. Once again, economic considerations will clarify the issue. To this purpose, the issuer's profile has to be sketched first.

6.3 Drawing the issuer's profile

The definitions of the hit rate, false alarm rate and ratio introduced in Appendix, Section 12.1. are applied again. As explained in Section 4 and Appendix 12.1, the false alarm ratio will be preferred to the false alarm rate. Being the ratio of the number of mistakenly issued warnings to the total number of issued warnings, the false alarm ratio describes better the quality of the service provided by the issuer. Furthermore, the number of non-events becoming large when high meteorological thresholds are considered, the false alarm rate dwindles accordingly. On the contrary, the false alarm ratio, remaining stable, happens to be a better estimator of the issuer's performance when extreme events selected with high meteorological thresholds are to be considered.

The results of simulations of up to 10.000 cases are presented in Figure 13. Meteorological thresholds are $Q = 8$ and 12 units, on the top row and $Q = 16$ and 20 units, in the bottom row. Yellow - orange curves describe relative operation characteristics computed with the false alarm ratio. Green - blue curves describe relative operation characteristics computed with the false alarm rate. All curves are parametrised in probability thresholds, running from $P = 5\%$ to $P = 95\%$ in steps of 10%. These are the probabilities sketched in Figure 12 and taken into account in the decision scheme given in Section 6.2.

Unmistakably do green curves telescope onto the left edge of the diagrams when higher meteorological thresholds are considered. On the contrary, orange curves are stable, remaining glamourously settled at centre stage when meteorological thresholds are high, thus demonstrating the better reliability of the false alarm ratio when extreme events are at stake. In each diagram, the yellow broken line with red vertexes represents the rough results directly emanating from the simulation. The orange continuous curve is obtained after polynomial smoothing of the broken line. The broken line behaves fairly regularly when low meteorological thresholds are considered (top row in Figure 13), and happens to be seriously shaken by higher meteorological thresholds (bottom row in Figure 13). Indeed, in the latter case only seldom and extreme events emanating from the right tail of the climatic profile (Figures 2 ,4 and 6) are taken into consideration, thus providing quite spare statistics.

6.4 Concluding remarks

Considering the analogy with the addressee's perspective discussed in Section 5.1.2, one notices that, for one given meteorological threshold Q, low probability thresholds correspond to high hit rates and false alarm ratios, respectively high probability thresholds to low hit rates and false alarm ratios.

According to the characterisation of risk awareness presented earlier, the addressee, behaving rationally, *de facto* adopts risk adverse strategies when warnings are issued at low probability thresholds, comparatively risk friendly strategies when they are issued at higher probability thresholds. Correspondingly, issuers exhibit risk friendly behaviours when they deliver their warnings at low probability thresholds, thus tolerating high false alarm ratios.

On the one side do risk adverse addressees or customers require to be alarmed early, at low probability thresholds. On the other side, risk friendly issuers tend to deliver their warnings at low probability thresholds either. This apparent contradiction will be given more attention in Section 8, Intermezzo.

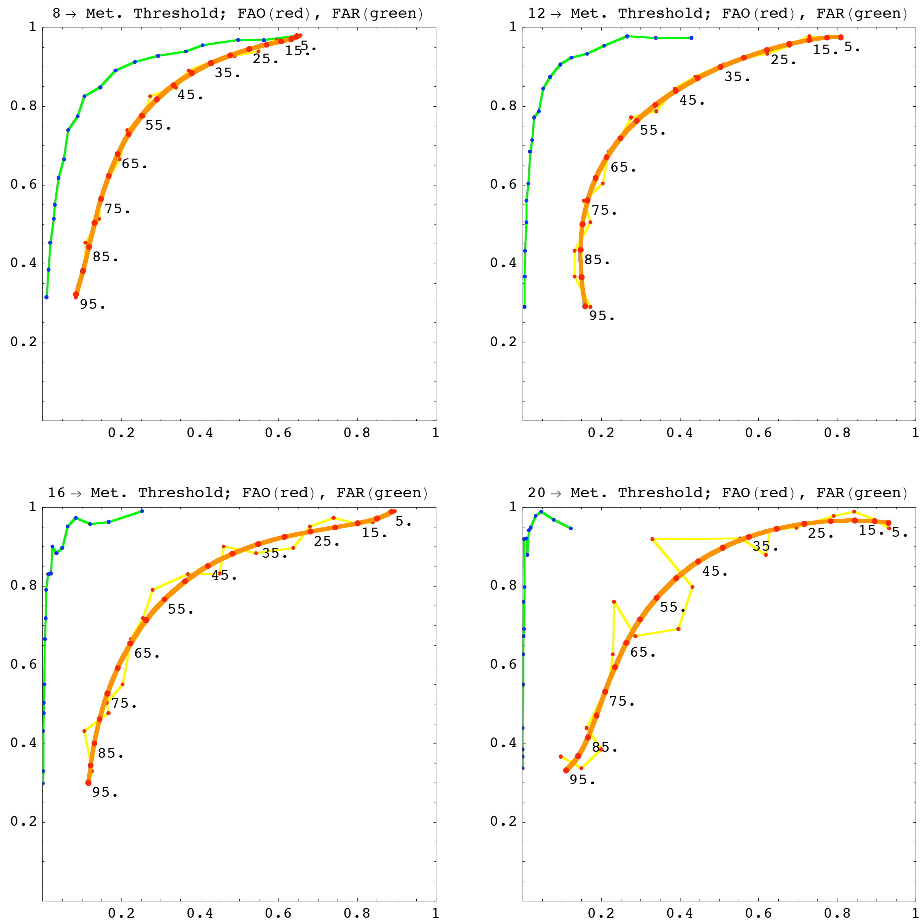

Figure 13: *Relative operation characteristics derived from simulations of the EPS. Abscissa: false alarm ratio (rate); ordinate: hit rate. Orange / yellow curves describe the Fao-ROC, green curves the Far-ROC. The parametrization running on each curve expresses the corresponding probability threshold. Top row: meteorological thresholds 8 and 12 units. Bottom row: meteorological thresholds 16 and 20 units.*

7 Synthesis: Warning Decision

Having established both actors' profiles, the next step will consist in confronting them. However, the addressee's profile being expressed in hit rate and false alarm rate, the issuer's one in hit rate and false alarm ratio, a reformulation of the former is required in order to implement it into the latter's related frame. This operation is described first. The computation of the optimal probability threshold at which warnings should be issued will be addressed in a following Section. Concluding remarks close the Section.

7.1 Implementing the rational addressee's profile

The elements presented in Section 5.2 remain valid and are repeated here: L represents the loss induced by a disaster for which neither a warning was issued nor mitigating measures were taken. C represents the costs induced by mitigating measures. They are due in case of occurrence of a correctly warned event, as well as in case of a mistakenly warned non-event. λ represents residual disaster costs remaining in the case of a correctly warned event.

	no event	event
warning issued	C	$C + \lambda$
no warning issued	0	L

Table 3.

The average costs the addressee is faced to during a period long enough to be of climatological relevance (expression (5)) are:

$$M = \frac{1}{a+b+c+d}\left[bL + Cc + (C+\lambda)d\right] \tag{8}$$

with a, b, c, d, Hr and Fao as defined in Appendix, Section 12.1 and computed following the decision scheme presented at the end of Section 6.1.

The derivation of the economic function based on hit rate and false alarm ratio is presented in the Appendix, Section 12.5. This function, hereafter referred to as M_A, reads:

$$
\begin{aligned}
M_{A(Hr,Fao)} &= \Omega L \, [(1 - Hr) + Hr \, \Lambda + \Gamma \, \frac{Hr}{1 - Fao}] && (9) \\
&= \Omega L \cdot \begin{bmatrix} 1 & \Lambda & \Gamma \end{bmatrix} \centerdot \begin{bmatrix} 1 - Hr \\ Hr \\ Hr \, (1 - Fao)^{-1} \end{bmatrix} && (10)
\end{aligned}
$$

It can be written either as a standard algebraic expression (9), or as a scalar product (10). Disentangling the roles played by the different protagonists, the second set-up, equation (10), will be preferred for reasons explained below in this Section.

The ***climatic burden*** ΩL represents the average costs the addressee would face if he were to assume his climatic fate without undertaking any mitigating or protective action when adverse weather events occur. As an example, were he expecting adverse weather events costing $L = 1000$ monetary units with probability of occurrence $\Omega = 0.1$ per year, then, over a long period of time, he would have to pay a yearly climatic burden of 100 monetary units.

However, proactive players being considered in this study, it will be assumed that the addressee decides to rely on warnings provided by an issuer, therefore requiring his actual burden to lie well below the "fateful" climatic burden. Explicitly, he will seek for the minimum between those two quantities, ΩL and $M_{A(Hr,Fao)}$, the first one being governed by the climate, the second one by the performance of the warning system. Accordingly, the economic function is modified into:

$$
\mathcal{M}_{A(Hr,Fao)} = \Omega L \cdot Min\{1 \; , \; \begin{bmatrix} 1 & \Lambda & \Gamma \end{bmatrix} \centerdot \begin{bmatrix} 1 - Hr \\ Hr \\ Hr \, (1 - Fao)^{-1} \end{bmatrix} \}
$$

Moreover, considering that an insurance company shrewdly hedging its risks could offer to cover the addressee's financial exposure at a premium $\mathcal{P}$ set well below the climatic burden ΩL, the issuer would enter into competition with that company and, for the addressee, the comparison would no longer occur between the climatic burden and the issuer's service, but instead between the contract presented by the insurance company and the service offered by the issuer. In such a situation, the coefficient 1 in the $Min\{1, \cdots\}$ expression would have to be replaced by the "insurance coefficient" $\mathcal{S} = \frac{\mathcal{P}}{\Omega L} \leq 1$.

The rational addressee's profile is finally implemented in accordance to equation (7): $\Gamma_{(Q)} = (1 - \Lambda)\, E_{(Q)}$. Then the economic profile at meteorological threshold Q, expressed in monetary units, reads:

$$\mathcal{M}_{Q(Hr,Fao),\mathcal{S}} = \Omega L \cdot Min_{Q,(Hr,Fao),\mathcal{S}} \tag{11}$$

with:

$$Min_{Q,(Hr,Fao),\mathcal{S}} = Min\{\mathcal{S}\ ,\ \begin{bmatrix} 1 & \Lambda & (1-\Lambda)E_{(Q)} \end{bmatrix} \cdot \begin{bmatrix} 1 - Hr \\ Hr \\ Hr\,(1 - Fao)^{-1} \end{bmatrix} \}$$

The promised disentanglement can now be explained: four actors are being taken into consideration in the above expression. They are: 1) the climatic burden ΩL, 2) the insurance company, whose possible action is parametrized by $\mathcal{S}$, 3) the addressee's profile, described by the row vector $\begin{bmatrix} 1 & \Lambda & (1-\Lambda)E_{(Q)} \end{bmatrix}$, and 4) the issuer's performance profile, described by the column vector $\begin{bmatrix} 1 - Hr & Hr & Hr(1 - Fao)^{-1} \end{bmatrix}^T$, where T means "transposition". Remarkable is the fact that the cost-loss ratio does no longer explicitly appear. It has been made implicit by the requirement of rationality. The possible intervention of an insurance company being ignored in the following, the coefficient is accordingly set to: $\mathcal{S} = 1$ for all considerations and in all graphs presented hereafter.

The graph of function (11) is presented in Figure 14 for an addressee operating at meteorological threshold $Q = 20$ units. The plateau visible on the rear left side of the surface corresponds to the area where, the false alarm ratio being high, the reliability of the warning system is poor and the costs induced by numerous false alarms are higher than the climatic burden ΩL. This function will be instrumental in the determination of the appropriate hit rate and false alarm ratio at which the warning system should be operated. This will be done in the next Section by the computation of an optimal probability threshold P^*.

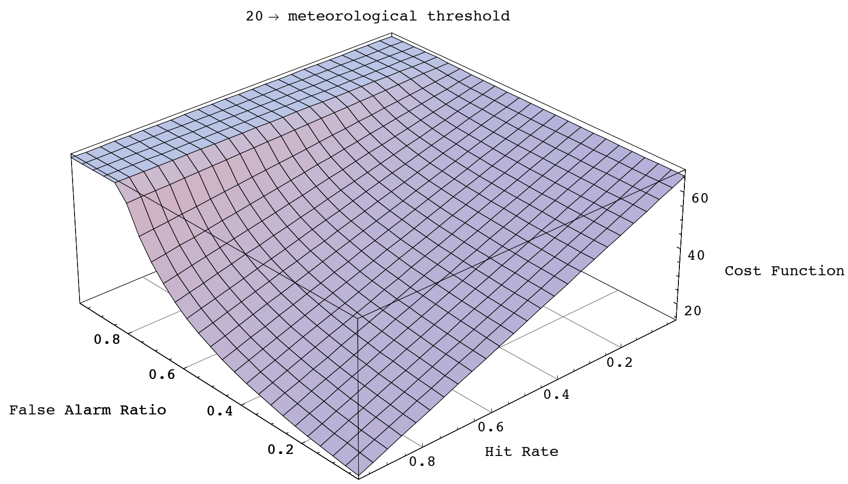

Figure 14: $\mathcal{M}_{Q(Hr,Fao)}$, *representing the economic function. The hit rate runs along the right edge of the cube, the false alarm ratio along the left edge. The meteorological threshold is arbitrarily fixed at* $Q = 20$ *units. The plateau visible on the rear left side corresponds to high false alarm ratios and poor issuer's reliability. Indeed, the addressee is likely to deal with an insurance in this is the area.*

7.2 Optimal probability threshold P^*

Profiles of the rational addressee and of the issuer being now both expressed in terms of hit rate and false alarm ratio, they can be superposed onto the corresponding ROC(Hr,Fao)-frame $\{[0,1] \times [0,1]\}$, as presented in Figure 15, indeed the exact equivalent of the panels shown in Figure 13. The network of black lines depicts the iso-costs of the rational addressee's economic function $\mathcal{M}_{Q(Hr,Fao)}$ introduced in Section 7.1 and presented in Figure 14. On each panel in Figures 15 and 16, the orange ROC-curve describes the issuer's profile, the green curve the rational addressee's profile, all of them being computed at a specified meteorological threshold Q^*.

Following the same methodology as in Section 5.3, it is considered that the addressee will seek out his minimum financial burden in the long term, and will therefore require to be warned at that probability threshold providing the minimum value of the economic function $\mathcal{M}_{Q^*(Hr,Fao)}$. Noticing that the rational addressee's profile is convex and the Issuer's profile concave, it appears that the optimal probability threshold is naturally defined at the point of tangency of both profiles, represented by the green dot in Figure 15 and on each panel in Figure 16. Thanks to the convexity of the iso-costs and the concavity of the ROC, it is that unique point of the ROC where the economic function reaches its minimum. Then the determination of the optimal probability threshold P^* is based again on the risk / reward approach used in Section 5.3. This time, however, rational addressee's and issuer's profiles are taken into consideration:

$$\frac{\partial_P Hr}{\partial_P Fao}|_{Issuer_{(Q^*)}} = \frac{\partial Hr}{\partial Fao}|_{Addressee_{(Q^*)}}$$

No longer as straightforward as the case discussed in Section 5.3, the derivation is presented in the Appendix, Section 12.6, and leads to the determination of P^*. The result is geometrically presented in Figures 15 and 16, the latter for various meteorological thresholds. Yellow curves describe issuer's, green curves addressee's profiles. Yellow curves are relative operation characteristics, green curves iso-costs. As already evoked in Section 5.5, iso-costs can be interpreted as addressee's equal sensibility or iso-tolerance loci.

Besides the climatic burden defined earlier, the ***warned burden*** the addressee is exposed to when operating at meteorological and probability thresholds Q^* and P^* can now be precisely defined. It is the monetary valuation at location $\{Hr_{(P^*)}, Fao_{(P^*)}\}$ of the economic function given by equation (11): $\mathcal{M}_{Q^*(Hr_{(P^*)},Fao_{(P^*)})}$. Together with the climatic burden, it will play a pivotal role in the estimation of the impact of the warning system. This issue will be discussed in Section 9.

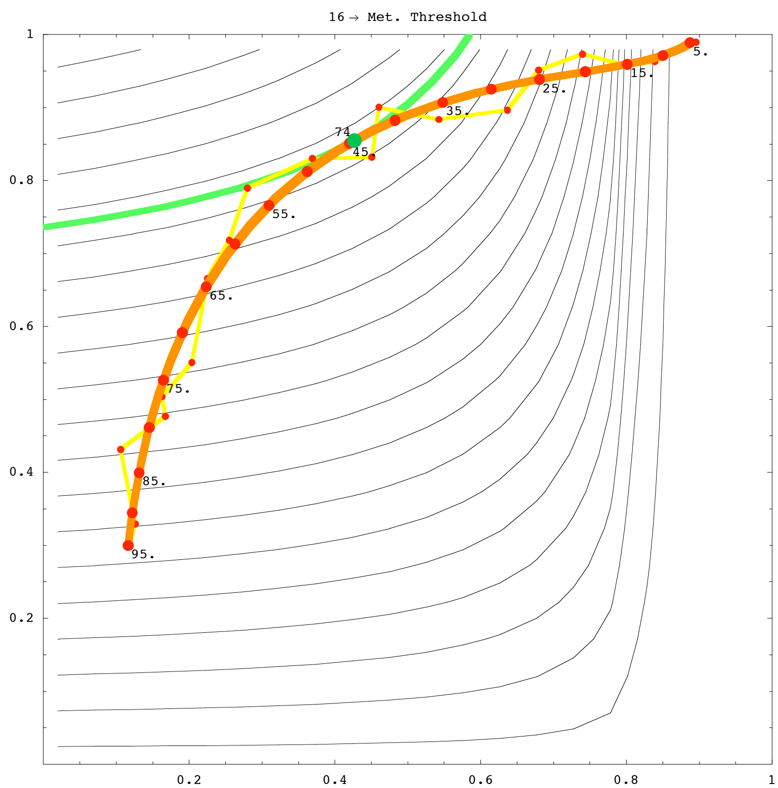

Figure 15: *Issuer's (orange) and rational addressee's (green) profiles. Abscissa: false alarm ratio; ordinate: hit rate. Meteorological threshold $Q^* =$ 16 units with the corresponding optimal probability threshold represented by the green dot at probability $P^* = 44$ %. The network of black lines represents the iso-costs of the economical function (Equation 11, Figure 14). The "74" plotted near the tangency point figures an efficiency measure expressed in %, to be discussed in Section 9.*

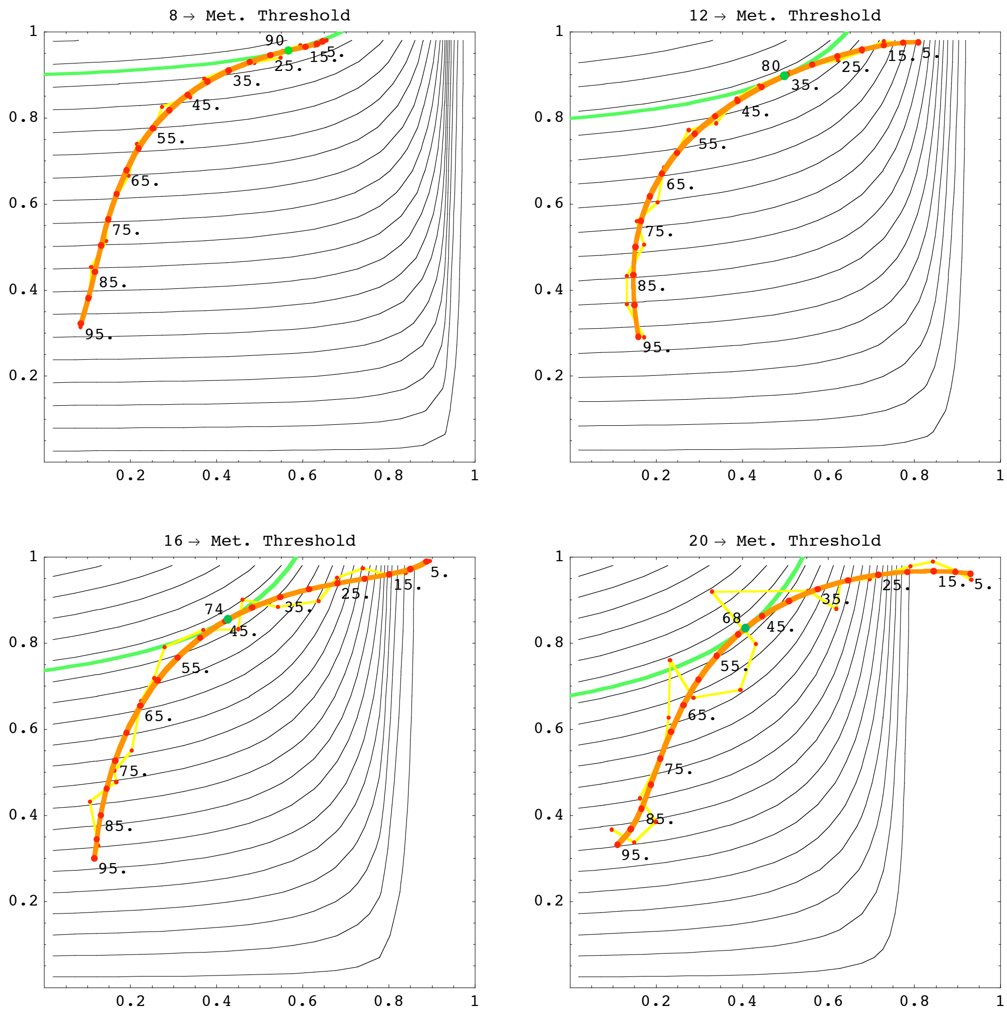

Figure 16: *Issuer's (orange) and rational addressee's (green) profiles as in Figure 15 for meteorological thresholds $Q^* = 18$, 12, 16 and 20 units.*

7.3 Concluding remarks

As revealed in the Appendix, Section 12.6, all economic parameters, i.e. costs, losses, residual losses, as well as the deduced parameters Γ and Λ, disappear in the course of the computation of the optimal probability threshold P^*. Having served as scaffoldings in the elaboration of the relationship between both actors, they vanish in the derivation, eventually leading to a relationship connecting exclusively the forecasting skill of the issuer and the disaster profile of the addressee (Section 5.1.1, equation 2, Figures 3 and 4). The model is focused onto the issue at stake, risk management. This is an amazing outcome.

A pragmatic relationship can nevertheless be established between the addressee's economic parameters, on the one side, and the performance achieved by the warning system, on the other side. Derived in the Appendix, Section 12.6, it reads:

$$\frac{\Gamma}{1-\Lambda} = \frac{(Fao-1)^2}{Hr\frac{\Delta Fao}{\Delta Hr} - Fao + 1} \tag{12}$$

Formally equivalent to equation (7), its interpretation is made clear with an example. Let we suppose that a warning system operates at hit rate and false alarm ratio $[Hr = 0.8, Fao = 0.4]$ with $\frac{\Delta Fao}{\Delta Hr} = 3$. (this last figure means that the slope of the ROC at the operating point $[Hr = 0.8, Fao = 0.4]$ is 1/3). The ratio $\frac{\Gamma}{1-\Lambda}$ can then been evaluated and gives 12%. Thus an approximation of the addressee's cost loss ratio is directly derivated from the issuer's performance. It is exact if the residual loss ratio Λ is known.

Finally, comparing the expression relating the addressee's risk profile to his economic profile, developed in Section 5.3:

$$\frac{\partial_Q Hr}{\partial_Q Far}|_{Riskprofile} = \frac{\partial Hr}{\partial Far}|_{Economicprofile}$$

with the corresponding expression relating the issuer's profile to the addressee's, developed in Section 7.2 :

$$\frac{\partial_P Hr}{\partial_P Fao}|_{Issuer_{(Q^*)}} = \frac{\partial Hr}{\partial Fao}|_{Addressee_{(Q^*)}}$$

and further daring to identify the addressee with his economic profile, one notices that the weather related risk the addressee is confronted with, expressed as risk profile, is transformed into a decision related risk, borne by the issuer. This decisional risk is expected to be low if the performance of the warning system is high. In any case, it should be lower than the risk induced by unanticipated adverse weather events.

The ability of the model to translate weather related risk into decision related risk is another startling outcome of this study.

8 Intermezzo: connections with finance and media

Relationships with the finance and media worlds are briefly discussed in a relatively informal style in the following Sections. On the one side, developments that occurred three decades ago in financial mathematics triggered the emergence of innovative lines of products and opened highly successful markets. On the other side, the multiplication of "warning providers" on the heavily crowded media driven stage requires a reflection related to the communicative impact of the services provided by official warning issuers. Both issues were inspirational in the elaboration of present work.

8.1 Comparison with the modern portfolio theory

Figure 17 sketches the "efficient frontier" (sometimes called the Markowitz frontier) considered in portfolio management. It is in startling agreement with the ROCs introduced in the previous Sections.

The expected return of an asset is plotted on the vertical coordinate, the standard deviation of this return, considered as the measure of the corresponding risk, on the horizontal coordinate. Every asset combination can be plotted in this return - risk space, and the collection of all such assets defines the region below the efficient frontier. Portfolios are constructed as combinations of individual assets. From a meteorological point of view, ensemble predictions can be assimilated to portfolios whose assets would be the members of these ensemble predictions. The analog of the addressee's profile in the financial world is called the Capital Allocation - or Market - Line.

Portfolios lying along the efficient frontier produce the highest return for a given risk or, alternatively, the lowest risk for a given return. The efficient frontier is frequently dubbed "the hedge" in financial jargon. Financial products lying on the hedge are said to be "hedged" and accordingly called "Hedge Funds". Correspondingly, inefficient portfolios and securities lie below the hedge. Thus, daring to close the analogy, optimal ensemble predictions might be compared with financial products. Best possible warning performances, instead of being provided "on the Hedge" would be settled "on the ROC". Of course, poor warning systems would accordingly be associated with inefficient portfolios and securities.

It is worth noticing that there exists no parametrization running on

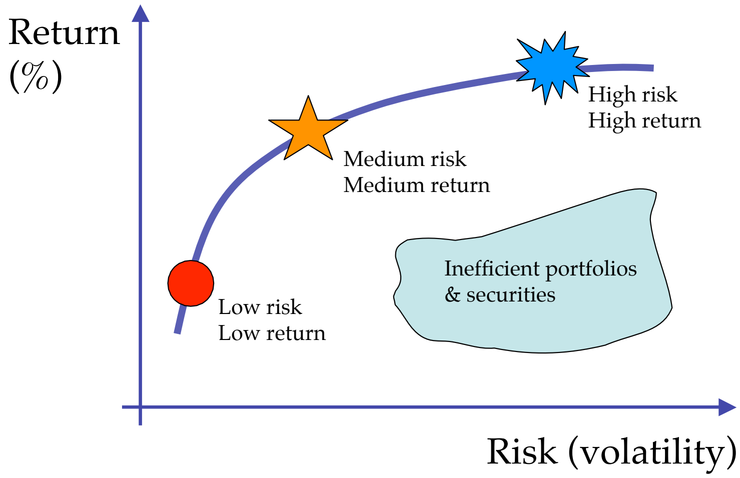

Figure 17: *Efficient Frontier as equivalent to the relative operation characteristics. Risk awareness expressed in the realm of the Hedge Fonds industry: Trade-off between risk and return. Adapted after F. S. Lhabitant. Reference [10].*

efficient frontiers. They simply express the relation between return and volatility. On the contrary, both ROCs considered here are parametrized. The first one, describing the addressee's profile, is expressed in terms of meteorological threshold, the second one, related to the issuer's profile, in terms of probability threshold. This dual parametrization is instrumental in the optimal tuning of the warning system.

8.2 Competition among warning instances

A striking difference between financial and meteorological realms becomes apparent when comparing Figure 17 and Figures 5/10. Risky portfolios, favoured by risk friendly investors, are located on the top right end of the efficient frontier, where risk adverse addressees have been settled in our meteorological setting. (Figures 5 and 10). Indeed, as already suggested, the behaviour of risk friendly investors should rather be compared with that of exuberant warning issuers who would tend to "overwarn". Such an attitude naturally emerges when several warning organisms operate on a competitive basis within a media driven society.

The ”Media forcing” arrow in Figure 18 depicts this trend. Research and development efforts resulting in improvements of the relative operation characteristics are sketched with the blue - red arrow. They are aimed at increasing the hit rate and reducing the false alarm ratio. Uncontrollable rivalry among risk friendly warning issuers pulls in another direction and conceals the risk of jeopardising the stability as well as the credibility of the warning system [Reference 11].

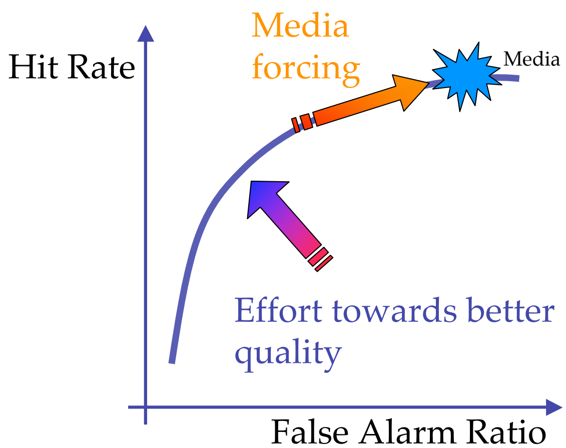

Figure 18: *Media forcing: warning issuers are inclined to operate at low probability thresholds and high false alarm ratios, thus jeopardising the credibility of a warning system.*

As a matter of fact, primarily risk adverse addressees inconspicuously tend to support this trend. An ultimate correction to this quite unfortunate retroaction would indeed consist in increasing their risk congeniality! They would then simply disregard premature warnings. Modifications on disaster profiles could improve the addressee’s resilience to adverse weather events as well, thus supporting their risk congeniality. On the issuer’s side, legal actions could help asserting the central position of national meteorological and hydrological services on the media driven stage. Both classes of actions are in political hands.

9 Performance versus Efficiency

Experience unveils divergences in the expectations laid by our actors on the warning system they are together bound to. The primordial addressee's requirement is efficiency. The warning system has to be tuned in a way keeping the disastrous impact of adverse weather events on his business at minimum. Costs induced by false alarms are to be considered as well. From the addressee's perspective, and following the concepts introduced so far, the efficiency is to be measured in monetary units (notwithstanding the fact that human distress can sometimes hardly be converted into such units). On the contrary, the issuer is genuinely interested in reaching the maximum performance of the system he is in charge of, thus preferably considering relative operation characteristics, hit rate, false alarm ratio, or any other valuation scheme of that kind.

The dualistic approach favoured in this study enables the elaboration of quantitative measures connecting both actors' expectations. Correspondingly, the assessing measures introduced in the following Sections consider the double perspective.

9.1 Assessing the performance of a warning system

The performance of the warning system has so far been considered as being given by the two parameters hit rate and false alarm ratio computed at meteorological and probability thresholds Q^* and P^*. Those thresholds having been determined for a specific addressee, the corresponding performance indexes, hit rate and false alarm ratio, are also specific to this addressee.

On the contrary, the "official" relative operation characteristic, noted "$\mathcal{ROC}$" and defined as

$$\mathcal{ROC} = \int_{Fao=0}^{Fao=1} Hr(f)df \tag{13}$$

is a global measure of the performance of a warning system, encompassing all probability thresholds and defined as the area measured below the ROC curve for false alarm ratios f running from $Fao = 0$ to $Fao = 1$.

Typographically, the curled "$\mathcal{ROC}$" designates in the following the performance of the warning system, indeed the surface under the "ROC", expression used throughout the document to represent the curve itself. It will be thereafter implemented as the issuer's performance measure and drawn on Figures 19 and 20 on the floor of the cubes as the surfaces encompassed by the yellow lines.

Finally, a technical remark: the ROC-curves having been established for probabilities ranging from 0.05 to 0.95, they are simply linearly extended from those values to zero and one in the computation of the $\mathcal{ROC}$s.

9.2 Assessing the efficiency of a warning system

In contrast to the previous case, the addressee's profile plays here a pivotal role. The efficiency measure proposed, whose derivation is presented in the Appendix, Section 12.7, is expressed as the ratio between two monetary quantities: $\mathcal{F} = \dfrac{\Delta_{warn}}{\Delta_{max}}$. Δ_{warn} is the monetary difference between the climatic and warned burdens, given by $\Omega L - \mathcal{M}_{Q,(Hr,Fao),\mathcal{S}}$. Δ_{max} is the maximum possible value of that difference.

Formally, the efficiency measure reads:

$$\mathcal{F}_{Q,(Hr,Fao),\mathcal{S}} = \frac{\Omega L - \mathcal{M}_{Q,(Hr,Fao),\mathcal{S}}}{\Omega L - \mathcal{M}_{Q,(1,0),\mathcal{S}}}$$

and depends on the hit rate as well as the false alarm ratio at a given meteorological threshold Q. Taking into account the presence of the $\mathcal{S}$ parameter, the eventual influence of an insurance company could be considered as well. Finally, a further simplification by ΩL gives:

$$\mathcal{F}_{Q,(Hr,Fao),\mathcal{S}} = \frac{1 - Min_{Q,(Hr,Fao),\mathcal{S}}}{1 - Min_{Q,(1,0),\mathcal{S}}} \tag{14}$$

According to this definition, the efficiency is zero when a warning system brings no positive departure in the addressee's financial burden from his climatic burden, or from the offer of an insurance company if $\mathcal{S} < 1$ (however, as mentioned in Section 7, $\mathcal{S} = 1$ in the present work). It equals one when the warning system, working perfectly, detects all events and issues no false alarms. The efficiency figures presented in Figures 19 and 20 have been computed by this way.

9.3 Interplay between performance and efficiency

Figure 19 illustrates the concepts yet introduced and reveals the interplay between performance and efficiency. In that figure and in Figure 20 the performance is measured on the horizontal bottom square, the efficiency in the vertical dimension.

The meteorological threshold Q^* is set again at 16 units. The issuer's profile and his ROC are drawn on the floor of the cube with the green dot representing the optimal probability threshold P^*, occurring at 44 %. The vertical dimension of the cube represents the difference $\Omega L - \mathcal{M}_{Q(1,0)}$ normalised between 0 and 1 following equation (13), with the climatic burden ΩL at the ceiling and the costs induced by a perfect warning system, $\mathcal{M}_{Q(1,0)}$, on the floor. The curtain vertically unfolded in space depicts the quantity $\mathcal{M}_{Q^*(Hr,Fao)} - \mathcal{M}_{Q(1,0)}$ normalised between 0 and 1 and computed on the ROC for (non-optimal) probability thresholds between 0.05 and 0.95. The correspondence of its minimum with the location of the probability threshold P^* eloquently testifies to the validity of the optimisation scheme implemented. Finally, the area encompassed by the yellow line on the floor expresses the $\mathcal{ROC}$, as defined in Section 9.1, equation (13).

The downward pointing blue arrow materialises the efficiency of the warning organisation $\mathcal{E}_{Q^*(Hr,Fao)} = 74\%$ at the optimal probability threshold $P^* = 44\%$. The red arrows drawn on the ceiling sketch the corresponding hit rate $= 85\%$ and false alarm ratio $= 43\%$. The two red dots glued at the ceiling reveal the fact that, the efficiency of the issuer being zero for those very low probabilities and very high false alarm ratios, the addressee should prefer to settle on an insurance contract.

The four cubes in Figure 20 illustrate the variation of the parameters presented in Figure 17 for meteorological thresholds Q^* set as usual at 8, 12, 16 and 20 units. The fall of the performance and the efficiency when higher meteorological thresholds are being considered rises the issue of the selection of an optimal meteorological threshold.

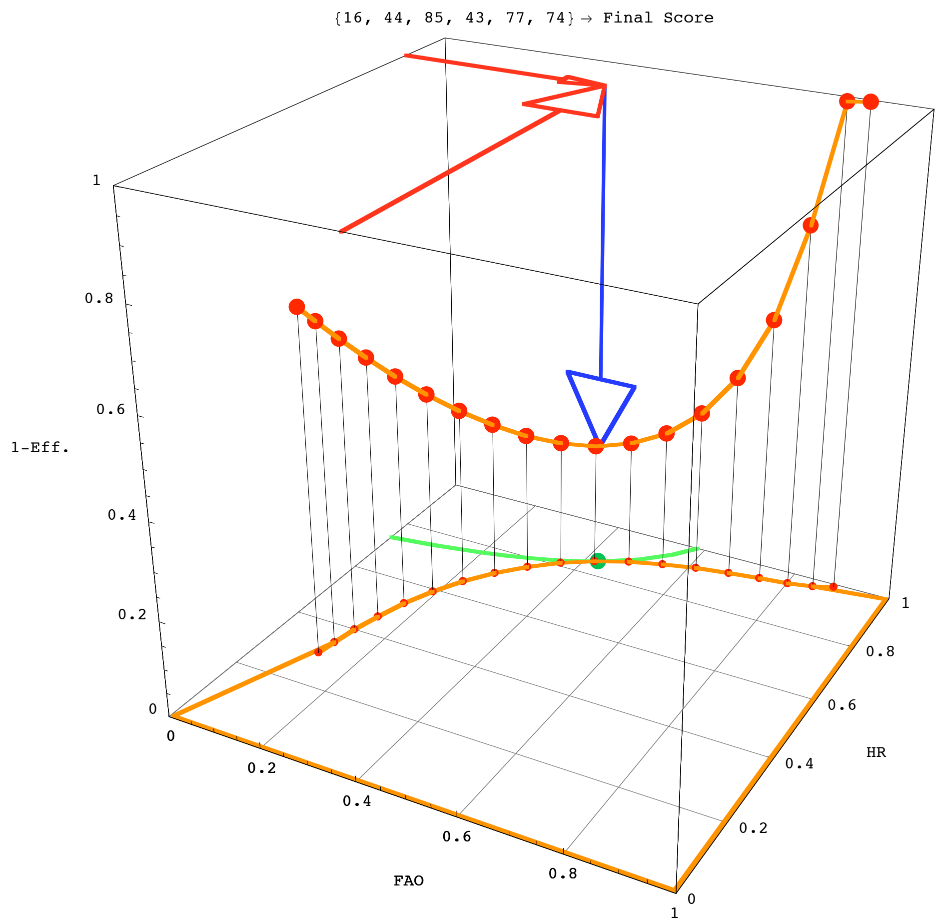

Figure 19: *Performance and efficiency indicators of the optimally tuned warning system. Abscissa: false alarm ratio; ordinate: hit rate; Vertical: normalised efficiency range (1-Eff) with minimum efficiency on the top, maximum efficiency on the bottom. Refer to text in present Section 9.3 for a comprehensive discussion. Figures given in the brackets are: meteorological threshold* = 16 *units, probability threshold* = 44%, *hit rate* = 85%, *false alarm ratio* = 43%, *relative operation characteristic* = 77% *and Efficiency* = 74%.

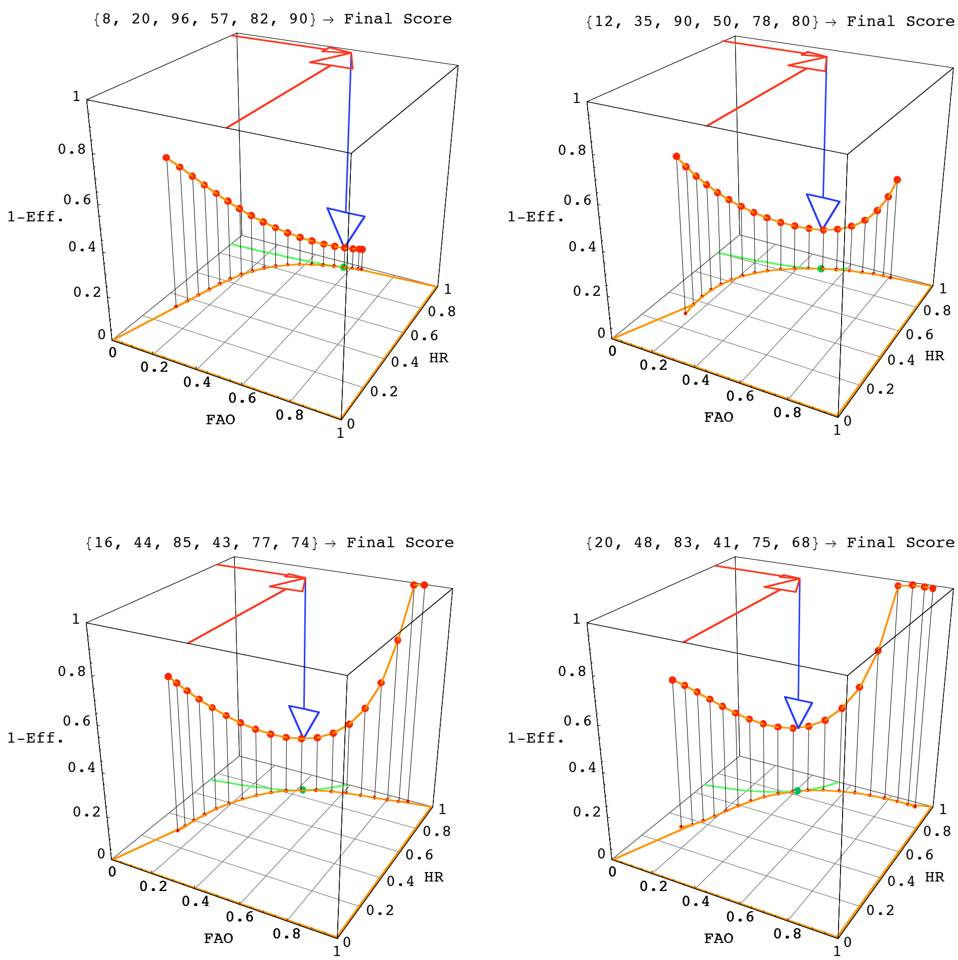

Figure 20: *Performance and efficiency indicators of the of warning system as in Figure 19, optimally tuned at thresholds* $Q^* = 8$*, 12, 16 and 20 units.*

9.4 Concluding remarks

The difference $\Delta_{warn} = \Omega L - \mathcal{M}_{Q(Hr,Fao)}$ between the climatic burden and the "warned burden", being expressed in monetary units, enables the valuation of the efficiency, or the impact, of the warning system onto the addressee's economic outcome. It is to be interpreted twofold. Firstly, it can be presented to the supervising ministerial body of a national meteorological service as a proof of the economic or societal impact provided by that meteorological service. Secondly, in case of commercial business, the fee to be paid in monetary units by the addressee to the issuer can be directly computed as a proportion expressed in percents of that difference: $Fee_{\{Addressee \longmapsto Issuer\}} = x\% \; \Delta_{warn}$. This way of proceeding is in perfect accordance with the use prevailing in financial business.

For those meteorological services following the tenets of the New Public Management (NPM), impact - or efficiency - rather than performance figures might be implemented into the operative agreements settled with their supervising ministerial bodies. Indeed, the measure of the impact a meteorological service exerts onto the community it is responsible for is in perfect accordance with the philosophy NPM is aimed at.

Furthermore, fee schemes based on efficiency might be used in the elaboration of the contracts or service level agreements established between the profit centres of that meteorological service and and their customers.

10 Financial and climatic interplay

The reader will have noticed that, for a given climate, the last parameter remaining free in the simulation is the meteorological threshold or, equivalently, the cost-loss ratio. They have to be defined by the addressee.

Two concealed degrees of freedom, not discussed until now, remain, however, open. All are to a certain extent in human hands. They are the tuning of the warning system, and the way both disaster and climate profiles might be shaped. Political decisions may either consist in modifying the disaster profile of a region, for example by constructing protective avalanches dikes, in improving the performance of the warning system, or even in undertaking both actions simultaneously. On the other hand, the climate has started its own journey. Whether humankind will be able to curb the track remains an open issue.

The first Section provides a methodology enabling the computation of optimal thresholds when disaster losses L are expressed in dependence of the intensity of incoming weather events, thus sketching the eventual financial outcome of political actions onto disaster profiles. Then, making use of this variable pricing model, the incidence of a modification of the climate profile on all parameters considered in the model is presented in the following Section. Considerations related to the possible closure of a warning system conclude the Section.

10.1 Variable Disaster Costs

The development has been conduced so far considering constant losses L, as introduced in Section 5.2, occurring in the case of a non-warned event, whatever its intensity. This is of course a simplifying hypothesis. Besides of having to expect a soaring frequency of disasters by increasing intensity of adverse weather, an aggravation in the financial impact per event has to be expected too.

The present Section is devoted to the formalisation of that dependency and to its subsequent implementation in the main framework. Thus, $L_{(Q)}$, expressed in monetary units and depending on $Q \in [0, B]$, is substituted to L. The consequences of that substitution, being subtle and multiple, have to be verified and assessed at multiple locations throughout the document. This thorough verification is presented in the Appendix, Section 12.8. The underlying concept is introduced in the following lines.

The simple climatic burden ΩL introduced in Section 7.1 is replaced by $\displaystyle\int_0^B C(q)E(q)L(q)dq$. This new expression integrates on the domain $[0, B]$ the frequency of occurrence $C(q)E(q)$ of weather induced disasters at intensity q, multiplied by the loss $L(q)$ engendered at that intensity.

Having this integral estimation of the climatic burden at hand, it is time to introduce the most dared artifice in the whole work: let us assume that, for the addressee, at least for his wallet, 10 disasters induced by one weather event and costing 10 monetary units each are equivalent to one disaster induced by that event and costing 100 monetary units. This allows us to introduce a reference disaster loss L_0 and a dimensionless equivalency factor $m_{(Q)}$ defined as the ***loss multiplier***. By this way: $L_{(Q)} = L_0\, m_{(Q)}$. Transported into the aforementioned integral, one has: $L_0 \displaystyle\int_0^B C(q)E(q)\, m(q)dq$. But, of course, the product $E(q)\, m(q)$ defines a new, more comprehensive disaster profile taking into account through the equivalence factor $m_{(Q)}$ the relationship between induced losses and event intensity. Let it be defined as $\mathcal{E}_{m(q)} = E(q)\, m(q)$. The extended climatic burden, taking variable disaster losses into account, is then given by:

$$L_0 \int_0^B C(q)\ \mathcal{E}_{m(q)} dq \tag{15}$$

One notices that this integral is formally identical to the integrals defining the hit rate and the false alarm rate, given by equations (2), (3) and (4), and in the definition of Ω. Variable disaster costs have simply been encapsulated into the newly defined $\mathcal{E}_{m(Q)}$ disaster profile. It is shown in the Appendix, Section 12.8, that all calculations performed so far remain valid under this substitution when the condition $\mathcal{E}_{m(Q)} < 1\ \forall Q \in [0, B]$ is satisfied.

Figure 21 presents an example of the profile of the equivalence factor $m_{(Q)}$ expressed in dependence of the intensity of weather events. In this case, disaster losses double when weather events reach intensity values of 12 units. The incidence of that operation onto the shape of the addressee's profile is presented in Figure 22:

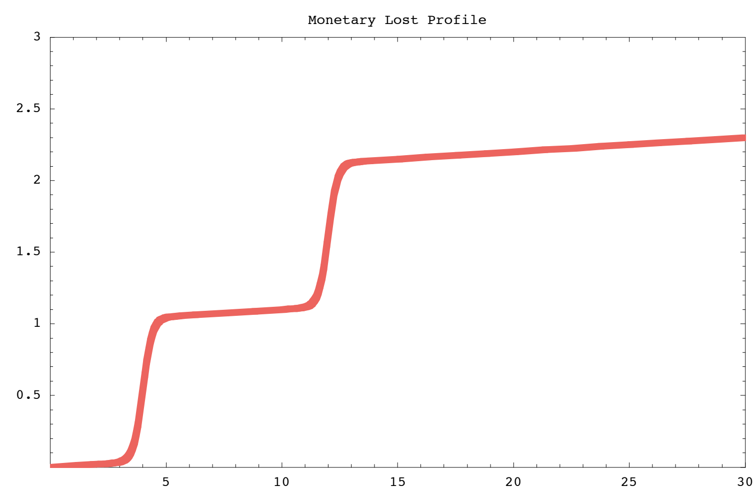

Figure 21: *Loss multiplier: example of the profile of a dimensionless function $m_{(Q)}$. It expresses the dependence of the disaster losses on the intensity of weather events. Abscissa: climate range; Ordinate dimensionless. Disaster losses are expressed as the product of a basic disaster loss L_0 expressed in monetary units and the multiplier: $L_0\, m_{(Q)}$. In this example, they are negligible below 5 units and double when weather events reach intensity values of 12 units.*

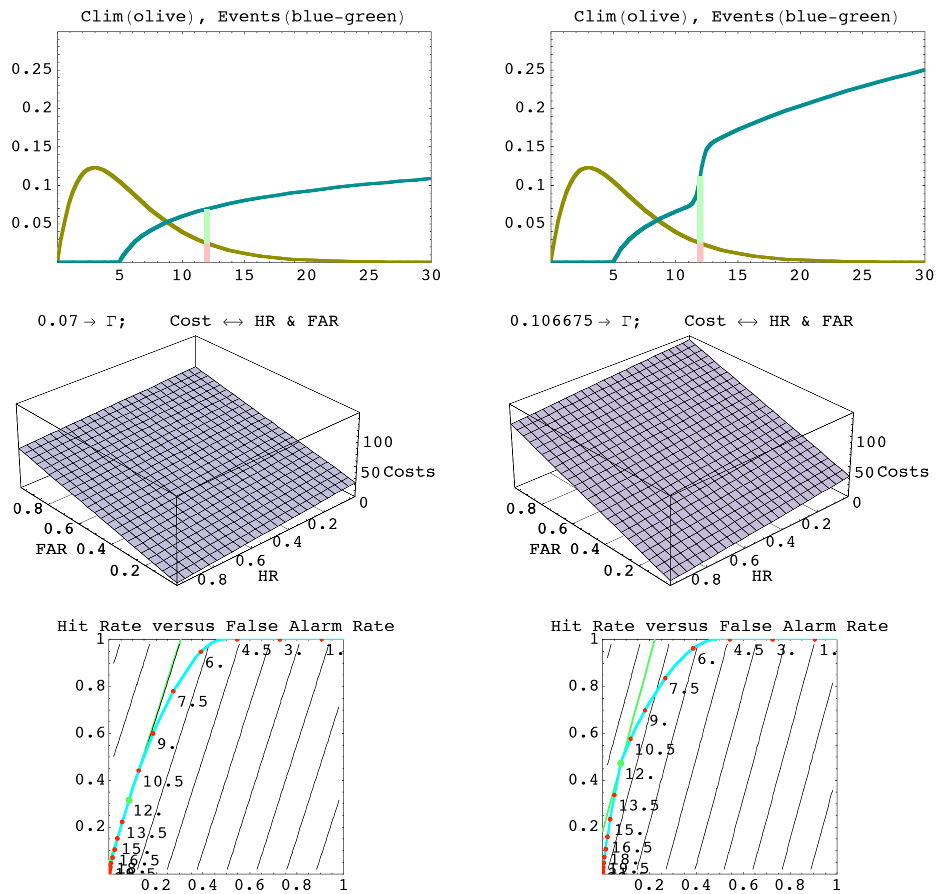

Figure 22: *Impact of the profile of monetary losses on the disaster profile. This Figure is the exact equivalent of Figure 11. However, the right addressee's disaster profile has been modified in accordance with the monetary losses function presented in Figure 21. In the third row, the difference between left and right rows is tiny. The shapes of the left and right ROC are slightly modified and the parametrization have been shifted. The slopes of the iso-costs, as well as the corresponding cost-loss ratios, remain, in this example, almost unchanged.*

10.2 Modified climatic profiles

All devices required to consider a modified climatic profile being, conceptually as well as computationally, available, the consequences induced by such a modification are easily simulated and straightforwardly presented in Figures 23 to 30. Two climate profiles are given in Figures 23 and 25. The first one has been used throughout the document and is referred as "reference climate" in the following. The second, modified climate profile, is simply shifted towards higher values of the meteorological parameter. It is indeed "warmed up" if this parameter happens to be a temperature. The area under both climatic curves is equal to one. It should be noticed that, besides the mean, the variance of the modified climate has increased too. The disaster profile defined in the previous Section remains unchanged with a meteorological threshold set to 14 units.

Figures 24 and 26 illustrate the impact induced by the climate change just simulated onto the performance and the efficiency of the warning system. One notices an improvement in the efficiency of the warning system from 72% to 79% by modified climate. Indeed, considering the evolution of the $\mathcal{ROC}$ from 76% to 84%, it appears that the improvement in efficiency can be related to the corresponding increase in the performance of the warning system.

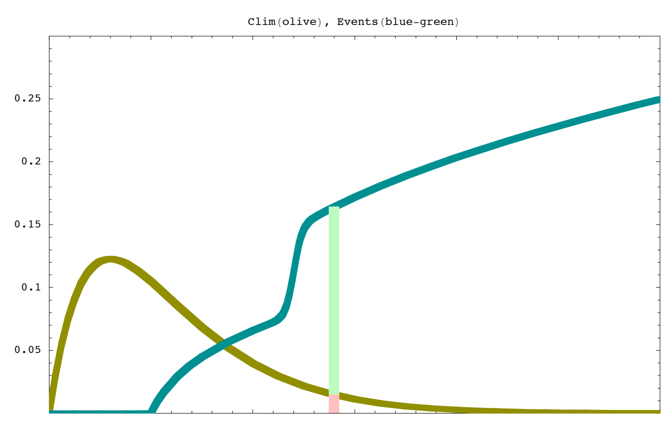

Figure 23: *Reference climate profile (olive curve) and disaster profile as defined in Section 10.1. Meteorological threshold set to 14 units. Abscissa: climate range; ordinate: probability of occurence.*

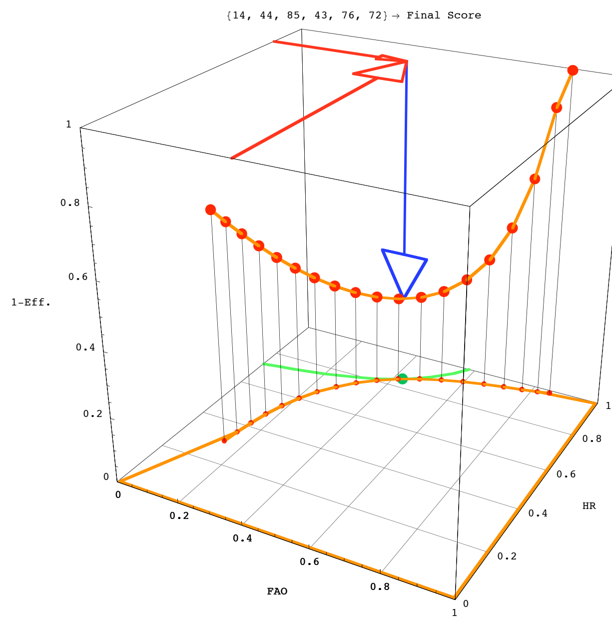

Figure 24: *Reference climate. Corresponding efficiency of the warning system.*

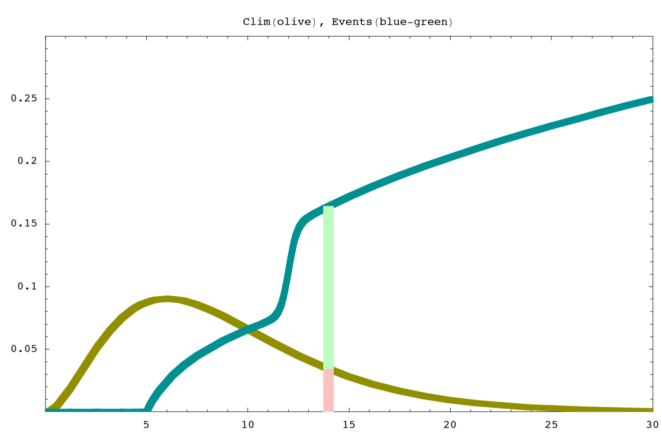

Figure 25: *Modified climate profile. As in Figure 23, but with a slightly "warmer" and broader climatic profile.*

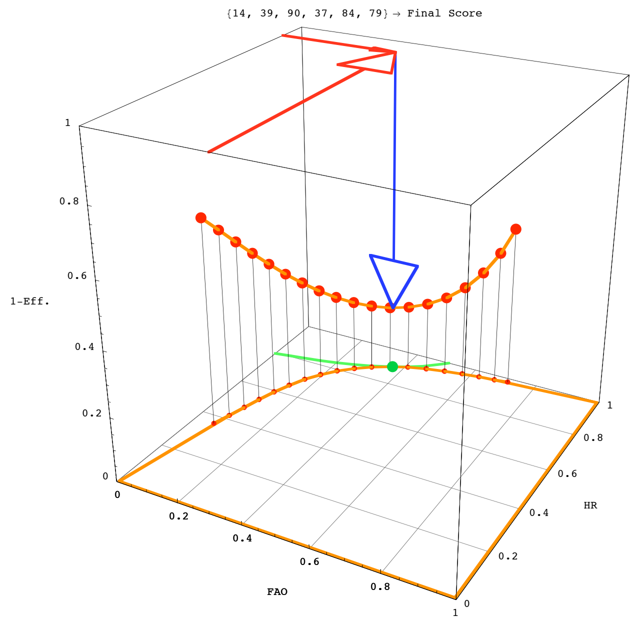

Figure 26: *Modified climate. Corresponding efficiency of the warning system.*

The question whether or not the meteorological threshold has been adequately chosen has to be raised. Figures 27 and 28 exhibit the variation of relative operation characteristics $\mathcal{ROC}$, efficiency and warned burden (average costs) for meteorological thresholds varying between 8 and 20 units, for both climate profiles. The warned burden has been divided by an appropriate factor in order to be scaled in proportion with the other curves.

The sensitivity of the warning in efficiency and performance (yellow and blue lines) to a modified climate is not substantial. This is due to the fact that the EPS simulator is tuned in order to be almost indifferent to the weather intensity. However, when the climate is modified, the warned burden (average financial burden: red lines) the addressee is confronted with rises significantly, in good proportionality with the disaster curve.

The interplay between the meteorological threshold the addressee has to choose, and the budget he has to devote to the mitigation of weather induced disasters is crucial and can now be demonstrated. Either will he determine a maximum budget for his mitigating actions, thereafter deducing the corresponding meteorological threshold, or he will define a convenient

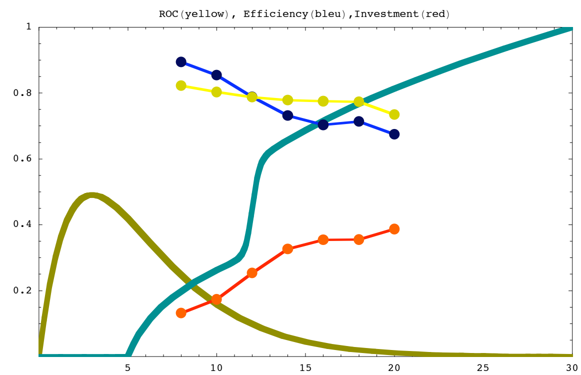

Figure 27: *$\mathcal{ROC}$ (yellow), Efficiency (blue), and warned burden (average costs, red) of the warning system. Reference climate profile.*

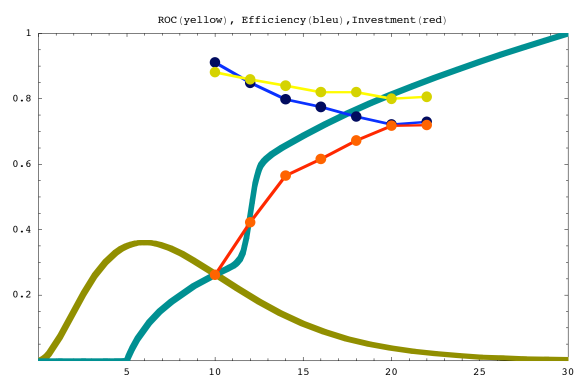

Figure 28: *$\mathcal{ROC}$ (yellow), Efficiency (blue), and warned burden (average costs, red) of the warning system. Modified, slightly "warmed up" climate profile.*

threshold, and then know his financial burden.

Following Table 4 reproduces all numerical figures computed in the examples presented so far. The figures given in the left and central columns emanate from the climate and disaster profiles given in Figures 23 and 24, and in the efficiency Figures 25 and 26 with $L_0 = 1000$ monetary units and $\Lambda = \frac{1}{20}$. The profile of monetary losses, (defined as the loss multiplier, Figure 21), is encapsulated in the expression $\mathcal{E}_{m(Q)}$ of the climatic burden. The figures in the three last lines of the table are expressed in monetary units. They represent the climatic burden, the warned burden and the financial burden that would be due if the model happened to be perfect.

Climate	reference	modified	modified
Parameter			tuned Q^*
Q^*	14 units	14 units	11 units
Γ	15.6 %	15.6 %	6.9 %
P^*	44 %	39 %	28 %
Hr	85 %	90 %	94 %
Fao	43 %	37 %	45 %
$\mathcal{ROC}$	84 %	86 %	86 %
$\mathcal{F}$	72 %	79 %	88 %
$L_0 \int_0^B C(q)\, \mathcal{E}_{m(Q)} dq$	33.09	66.6	66.6
$\mathcal{M}_{Q^*(Hr,Fao)}$	14.04	24.6	14.8
$\mathcal{M}_{Q^*(1,0)}$	6.8	13.7	7.9

Table 4.

The right column simulates the case where the addressee decides to operate under modified climatic conditions with almost the same warned burden $\mathcal{M}_{Q^*(Hr,Fao)} \approx 14$ monetary units as for the reference climate. In such a circumstance, having to modify his meteorological threshold from $Q^* = 14$ to $Q^* = 11$ units, he is indeed committed to increase his risk adversity. All these elements are reproduced in Figure 29. It must be emphasised that they are obtained with the warning system operating in background and letting the addressee trigger mitigating actions when the probability threshold is reached.

The impact of the climate change on the addressee's risk profile is figured by the modification on the ROC curves shown in the two bottom panels of Figure 30. As previously noticed, one observes on the bottom right panel

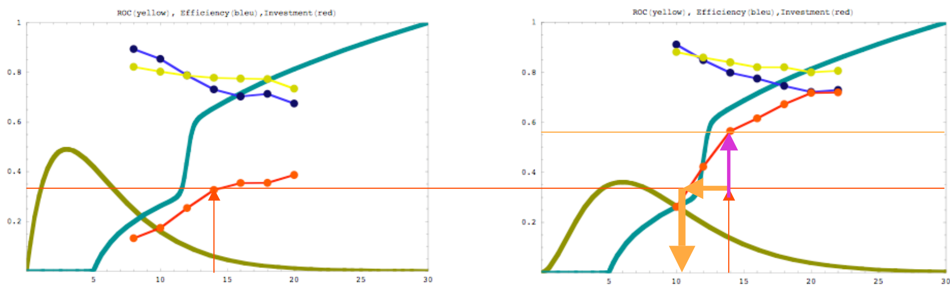

Figure 29: *ROC (yellow), efficiency (blue), and warned burden (average costs, red) of the warning system. Left panel: reference climate profile. Right panel: Modified climatic profile. The left panel corresponds to the left column in Table 4. On the right panel, the upward pointing pink arrow is related to the central column in the aforementioned table. The left and downward pointing yellow-orange arrows are related to the right column in the table.*

how the increase in risk adversity is figured by the reduction of the $\frac{\Delta Hr}{\Delta Far}$ slope. Furthermore, the leap in the disaster profile induced by the cost function (cost multiplier) produces the bend of the ROC-curve at threshold value = 12 units in the bottom right panel.

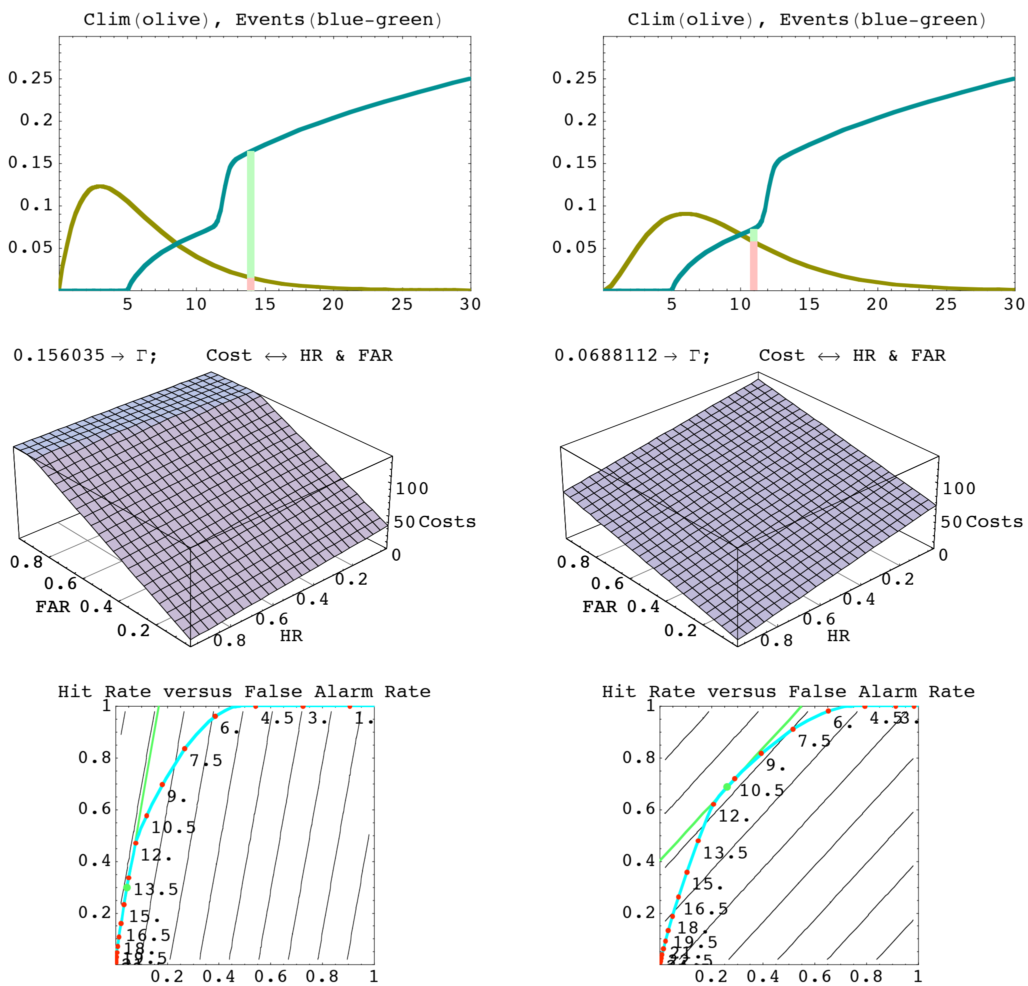

Figure 30: *Richardson function and risk profile of the addressee by modified climate.*

10.3 Concluding remarks

The system can now be considered as being closed onto itself. Both meteorological and probability thresholds are optimally tuned according to the constraints emanating from the interplay between the climatic profile, the disaster profile and the financial constraints the addressee is submitted to.

Some readers might be bewildered by the low probability thresholds that emerge from the computations. Compared with those operational services are acquainted with, they are indeed low. The origin of this fact is twofold. On the one side, the ensemble prediction simulator is tuned in order to produce fairly good probabilistic forecasts. On the other side, the disaster profiles implemented so far are indeed benign. A less skilled ensemble prediction system operated on harsher disaster profiles would require higher probability thresholds.

In any case, operational thresholds at which mitigating actions ought to be triggered should not be confused with statistical confidence indexes. Confidence indexes express trust in scientific hypotheses. Probability thresholds serve as basis in decision taking.

11 Conclusions

The first and foremost contribution of this study is conceptual. Accordingly, possible applications should have to be refined and tailored to specific addressee's requirements and issuer's capabilities. Such a course of action is not unusual. Considering for example the mathematical game theory or the capital asset pricing model in finance, devised decades ago, one notices that both theories opened avenues that first challenged, then transformed the way people understood the problems and issues they were confronted with. Algorithms, applications, services and products were introduced consecutively.

Operating on a infinitely more modest footing, the present work is aimed at clarifying the relationship between warning institutions on the one side and communities or organisations submitted to meteorological or climatic hazards on the other side. Inspired by the notion of duality that kindled the formal thinking in mathematics, physics and economics during the second half of the twentieth century, it is aimed at disentangling the addressee's and issuer's roles.

The first one, the addressee, is described by a disaster profile, measuring his exposure to adverse weather events, and an economic profile, assessing the monetary impact of those events. Both profiles are reciprocally asserted in a way enabling the computation of the meteorological threshold at and beyond which mitigating actions should be undertaken against threatening weather events. This reciprocal assertion enables the definitions of the addressee's rationality, as well as his risk awareness.

The issuer's profile is much more straightforward. It describes the performance of a warning system operating under the meteorological constraints occurring in the geographical area of relevance for the addressee. Forecasts are expressed probabilistically in terms of occurrence of weather events whose intensity lies at or beyond the meteorological threshold.

The confrontation of both addressee's and issuer's profiles then enables the computation of that optimal probability threshold at and beyond which mitigating actions should be triggered, according to the meteorological threshold.

A striking characteristic of the proposed model is its ability to directly relate the addressee's disaster profile to the issuer's performance profile.

Financial or monetary considerations, having served as scaffolding in the elaboration of the model, simply vanish at the the final stage of its set-up.

However, the methodology enables the definition of the addressee's climatic burden, the warned burden (average costs) he would be faced with if he were to operate his business without warning system and mitigating actions. Similarly, the average costs having to be paid when operating with a warning system can be evaluated. Thus the departure from the climatic burden to the warned burden provides a measure of the efficiency of the warning system. By this way, performance objectives defined on the issuer's side are translated into efficiency impact onto the addressee's business.

Simulation of modified climate profiles, on average as well as in spread, are introduced. Enabling the computation of optimal meteorological warning thresholds under modified climatic conditions, they provide either a measure of the financial impact of such climatic modifications on the addressee's business, or a valuation of a meteorological threshod by which the financial impact of a modified climate would be compensated. Both are considered under optimal tuning conditions of the warning system.

12 Appendixes

12.1 Basic definitions of hit rate, false alarm rate and Ratio

The classical contingency table describing the four possible occurrences where: a) neither an event occurred nor mitigating actions were undertaken, b) an event occurred without mitigating actions, c) mitigating actions were undertaken although no event occurred and d) an event occurred for which mitigating actions were correctly undertaken, is reproduced in Table A.1. The four figures a, b, c, d represent the number of cases accumulated in each category during an assessment period.

	no event	event	total
mitigating action undertaken	c	d	c+d
no mitigating action	a	b	a+b
total	a+c	b+d	a+b+c+d

Table A.1

The following definitions are well known: hit rate (also called Probability of Detection): $Hr = \frac{d}{b+d}$. false alarm rate (also called Probability of False Detection): $Far = \frac{c}{a+c}$. false alarm ratio (no other name): $Fao = \frac{c}{c+d}$. Frequency of occurrence of the event: $\Omega = \frac{b+d}{a+b+c+d}$.

As explained in the main text, besides the hit rate, which is of paramount importance for the addressee as well as for the issuer, the difference between the false alarm rate and the false alarm ratio is subtle and should not be underestimated. It is explained hereafter with the following example:

	no event	event	total
mitigating action undertaken	100	40	140
no mitigating action	850	10	860
total	950	50	1000

Table A.2

The false alarm rate, computed on "no events", in this case not so bad with $\frac{100}{950}$, measures the frequency at which the business of the addressee, instead of running smoothly, is impeded by unnecessary protective actions taken under fair weather conditions.

The false alarm ratio provides a measure of the quality of the service proposed by the issuer. It is expressed in terms of the number of mistakenly issued warnings divided by the total number of issued warnings, in this case with $\frac{100}{140}$ quite a poor score.

The false alarm rate is an addressee's issue, the false alarm ratio an issuer's concern. The former is a measure of the efficiency of a service, the latter a measure of the performance of that service.

12.2 Derivation of the probabilistic hit rate and false alarm rate

Taking into account the concepts introduced in Section 5.1.1, the contingency table is now formulated using the probabilistic distributions representing the climatology: $C(q)$, and the frequency of occurrence of disastrous events: $E(q)$, Table A.3:

Integration Domain	No event	Event occurs	Sum
$[Q, B]$	$\int_Q^B (1-E(q))C(q)dq$	$\int_Q^B E(q)C(q)dq$	$\int_Q^B C(q)dq$
$[0, Q]$	$\int_0^Q (1-E(q))C(q)dq$	$\int_0^Q E(q)C(q)dq$	$\int_0^Q C(q)dq$
$[0, B]$	$1-\Omega$	Ω	1

Table A.3

Being quite awkward, the interpretation of this table, formally corresponding to the tables introduced in Section 12.1, is accomplished with utmost care below. As a matter of definition, the term "case" used below refers to the lapse of time Δ defined in Section 5.1.1, lasting a hour, a day, even a week, during which a disastrous event might occur, or not. This disastrous event is exclusively triggered by the weather.

For a given case whose weather condition is Q, the probability of occurrence of a disastrous event is given by $E_{(Q)}$. The probability for the weather state to be equal to Q being determined by the climatic distribution $C_{(Q)}$, the probability of occurrence of a disastrous event triggered by the weather at value Q is therefore $E_{(Q)}C_{(Q)}$. Then, the integral $\int_{Q_1}^{Q_2} E(q)C(q)dq$ measures the probability to experience a disastrous event when the weather happens to occur between two meteorological bounds $Q_1 < Q_2$.

The upper row of the Table A.3, labelled $[Q, B]$, represents the cases at and beyond the warning threshold Q, for which mitigating actions are undertaken. The middle row of the table, labelled $[0, Q]$, represents those cases where the value of the meteorological parameter lies beneath the threshold and no mitigating actions are undertaken. The lower row of the table, labelled $[0, B]$, spans all the cases where mitigating actions are and are not undertaken.

Let us now chase in the table and consider first the entry $\{[Q, B]$, Disastrous events$\}$ of value $\int_Q^B E(q)C(q)dq$. This integral between Q and B measures the probability of occurrence of disastrous events within this domain, where mitigating actions are undertaken. Below, the integral $\int_0^Q E(q)C(q)dq$ measures the probability of occurrence of disastrous events within the complementary domain $[0.Q]$ in which no mitigating actions are undertaken. The sum of both, $\Omega = \int_0^B E(q)C(q)dq < 1$, measures the overall probability of occurrence of weather induced disasters. It represents the climatic component in the climatic burden ΩL discussed in the main text.

Considering now the entry $\{[Q, B]$, No disastrous events$\}$ of value $\int_Q^B (1-E(q))C(q)dq$, one notices that it can be written $\int_Q^B C(q)dq - \int_Q^B E(q)C(q)dq$. The first integral is the entry $\{[Q, B]$, Sum$\}$ representing the probability of occurrence of weather within the corresponding domain, with or without disastrous events. The difference between both integrals therefore measures the frequency of the cases when mitigating actions are undertaken although no disasters occur. Accordingly, the entry $\{[0, Q]$, No disastrous events$\}$ measures the frequency of the cases when neither mitigating actions are undertaken nor disasters occur.

The bottom row $[0, B]$ gives for both cases where disastrous events occur, or not occur, the corresponding sums expressed in terms of Ω. The bottom right corner is the sum on the last column as well as on the last row. It expresses the natural propriety of a probability: $\int_0^B C(q)dq = 1$.

Conclusively, the Table A.3 having the structure of a perfect square, according to their definitions given in Section 12.1, the two ratios can be directly read on it. They are:

Hit rate: expresses the ratio between the probability of disasters occurring in the domain $[Q, B]$, for which mitigating actions are undertaken and the overall probability of occurrence of weather induced disasters. Following the definitions given in Section 12.1 and taking into account the correspondence between Tables A.1 and A.3, $\int_Q^B E(q)C(q)dq$ is substituted for d and Ω is substituted for $b + d$, thus giving:

$$Hr_{(Q)} = \frac{1}{\Omega} \int_Q^B E(q)C(q)dq$$

False alarm rate: expresses the ratio between the probability of non occurrence of disasters in the domain $[Q, B]$ for which mitigating actions are inadequately undertaken and the overall probability of occurrence of weather conditions not triggering disasters. Following the definitions given in Section 12.1 and taking into account the correspondence between Tables A.1 and A.3, $\int_Q^B (1 - E(q))C(q)dq$ is substituted for c and $1 - \Omega$ is substituted for $a + c$, thus giving:

$$Far_{(Q)} = \frac{1}{1 - \Omega} \int_Q^B (1 - E(q))C(q)dq$$

The variable of these functions, the meteorological threshold Q, is an integration bound on the right hand side of each expression. This propriety will be given particular attention in following Section 12.3.

12.3 Derivation of the relationship between Γ, Λ and Q

The derivation is carried through as an optimisation problem. The economic profile represents the utility function to be minimised under the constraint provided by the risk profile, expressed as the relative operation characteristic. Both sides of the following expression therefore have to be evaluated:

$$\frac{\partial_Q Hr}{\partial_Q Far}|_{Riskprofile} = \frac{\partial Hr}{\partial Far}|_{Economicprofile}$$

Risk profile the propriety of the derivative of an integral with respect to its integration bound Q : $\partial_Q \int_Q^B f(q)dq = -f_{(Q)}$, is used here, yielding to:

$$\begin{aligned}
\frac{\partial_Q Hr}{\partial_Q Far}|_{Risk} &= \frac{\partial_Q[\frac{1}{\Omega}\int_Q^B E_{(q)}C_{(q)}dq\ ]}{\partial_Q[\frac{1}{1-\Omega}\int_Q^B (1-E_{(q)})C_{(q)}dq\ ]} \\
&= \frac{\frac{1}{\Omega}E_{(Q)}C_{(Q)}}{\frac{1}{1-\Omega}(1-E_{(Q)})C_{(Q)}} \\
&= \frac{E_{(Q)}}{1-E_{(Q)}} \cdot \frac{1-\Omega}{\Omega}
\end{aligned}$$

.

Of course, $E_{(Q)} < 1\ \forall Q \in [0, B]$ has to be satisfied and $\Omega \in [0, 1)$ assumed (Appendix, Section 12.2). In such circumstances, $\frac{\partial_Q Hr}{\partial_Q Far}|_{Risk} > 0$. Furthermore, $\partial_Q(\frac{\partial_Q Hr}{\partial_Q Far}|_{Risk}) = \frac{E'_{(Q)}}{(1-E_{(Q)})^2} \cdot \frac{1-\Omega}{\Omega} > 0$ for $E'_{(Q)} > 0$. These two requirements, $E_{(Q)} \in [0, 1)$ and $E'_{(Q)} > 0$, warrant the concavity of the ROC and therefore, together with the linearity of the iso-costs, the unicity of the solution of the optimisation problem.

Economic profile Taking into account the continuity of the Richardson function with respect to its variables Hr and Far, the theorem of the implicit function is applied in the sense that the iso-costs of the Richardson function (6) are simply its implicit functions. On any iso-cost of monetary value $M_\$$ the relation $M_R(Hr, Far) = M_\$$ holds for the corresponding Hr and Far values. Thus, if Hr is the implicit function expressed in terms of Far, the total derivative is:

$$D_{Far}M_R(Hr_{(Far)}, Far) = \frac{\partial M_R}{\partial Hr}\frac{\partial Hr}{\partial Far} + \frac{\partial M_R}{\partial Far} = 0.$$

Therefore, after few algebraic manipulations, the result reads (with "Eco" standing for "$Economicprofile$"):

$$\begin{aligned}
\frac{\partial Hr}{\partial Far}|_{Eco} &= -\frac{\partial M_R}{\partial Far}(\frac{\partial M_R}{\partial Hr})^{-1} \\
&= \Gamma L \frac{\Omega - 1}{\Omega(\lambda + L(\Gamma - 1))} \\
&= \frac{1-\Omega}{\Omega} \cdot \frac{\Gamma}{1 - \Lambda - \Gamma}
\end{aligned}$$

Synthesis Requiring the identity of the slopes at the point on the ROC with curvilinear abscissa Q and accordingly equating the two quantities:

$$\frac{\partial_Q Hr}{\partial_Q Far}|_{Risk} = \frac{\partial Hr}{\partial Far}|_{Eco}$$

leads to:

$$\frac{E_{(Q)}}{1-E_{(Q)}} \cdot \frac{1-\Omega}{\Omega} = \frac{1-\Omega}{\Omega} \cdot \frac{\Gamma}{1-\Lambda-\Gamma}$$

and after few further manipulations to:

$$\Gamma_{(Q)} = (1-\Lambda)\, E_{(Q)}$$

Indeed are these concepts embedded in the broader realm of the Lagrange Multipliers and issues related to constraint optimisation. Fortunately, in the present setting the derivation of the relationship between Γ, Λ and Q is simple and does not require the mobilisation of such a sophisticated weaponry.

12.4 Conceiving an EPS-Simulator

The methodology the simulator is based upon proceeds from a logical inversion. Weather events whose distribution follows a defined climatic profile are produced first, then the probabilistic forecasts, which are correlated to those events, are simulated. Indeed, instead of following the classical *ex ante* way peculiar to weather forecasting, an *ex post modus operandi* is applied. First should the event be generated, then the corresponding forecast that would have been produced by an Ensemble Prediction System will be simulated.

Precisely, the algorithm proceeds through the following steps:

Generate a meteorological event Starting from the climatic distribution defined in Section 5.1.1, generate an event of value $\tilde{Q}$. That event, represented by the downward pointing thick arrow in Figure 12, is a realisation of the random variable following the given climatic distribution, in this case a $\Gamma_{(r,s)}$ distribution with parameters $r = 2, s = 3$.

Construct the corresponding probabilistic forecast In the simple approach chosen here, probabilistic forecasts follow normal distributions $N(\tilde{Q} + \mu, \sigma)$ whose departure μ to $\tilde{Q}$ and variance σ are themselves produced by random generators: μ follows another normal distribution $\mu = N_{(t,u)}$ with $t = 0, u = 2$ and σ follows another Γ distribution $\sigma = \Gamma_{(v,w)}$ with $v = 2, w = 2$.

Repeat the two steps as long as needed in order to produce a file $< event, forecast >$ from which statistics will be established.

The whole system is simply formed by a cascade of three levels of random generators. It is worth repeating that the probability of occurrence applied to the simulated forecast corresponds to climatic exposure the addressee is confronted with: the simulated forecasting model experiences the same climate as the addressee. Another point is that neither a temporal reference nor a forecast term is considered. However, a fading in the forecasting skill of the model can be simulated in modifying and/or increasing the parameters t, u, v, w introduced above. As a matter of fact, together with the two random generators $N_{(t,u)}$ and $\Gamma_{(v,w)}$, these four parameters govern the proprieties of the simulated model.

Three simulations are presented in Figure 31. The meteorological threshold is arbitrarily set to $Q = 10$. Meteorological events $\tilde{Q}$ are sketched by the vertical blue bars, the corresponding simulated EPS-forecasts by the the yellow distributions. No warnings should be issued when blue bars fall into the green domain, up to Q, but ought to be in the pink domain. Probability of occurrence is measured in terms of the surface in pink domain below the yellow distribution and reported in black % on the panels. Were for example a probability threshold set at $P = 0.6$, then the first panel would represent a successfully warned event, the second one a correctly not warned non-event and the third one a missed event. Depending on the value of the meteorological threshold, up to 10.000 such events have been simulated in order to elaborate satisfying issuer's ROC Curves.

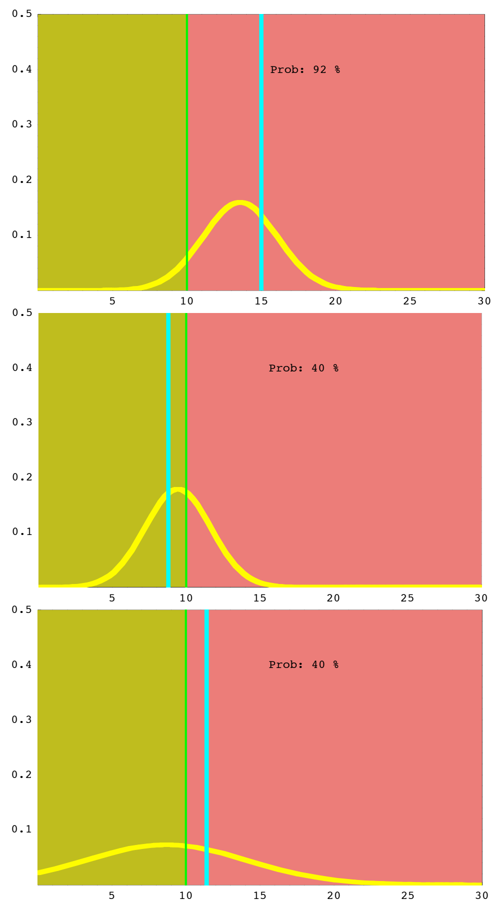

Figure 31: *Simulation of the EPS. The meteorological threshold is arbitrarily set to* $Q = 10$*. Meteorological events* $\tilde{Q}$ *are sketched by the vertical blue lines, simulated EPS-forecasts by the the yellow distributions.*

12.5 Derivation of the functions $M_{R(Hr,Far)}$ and $M_{A(Hr,Fao)}$

The addressee's economic profile can be expressed either in terms of hit rate and false alarm rate, or of hit rate and false alarm ratio. Both derivations are presented hereafter with a, b, c and d defined in Section 12 with the frequency of occurrence of meteorological events $\Omega = \frac{b+d}{a+b+c+d}$. The hit rate is given for both cases by $H = \frac{d}{b+d}$, the false alarm rate by $Far = \frac{c}{a+c}$ for $M_{R(Hr,Far)}$, the false alarm ratio by $Fao = \frac{c}{c+d}$ for $M_{A(Hr,Fao)}$.

12.5.1 Derivation of $M_{R(Hr,Far)}$

The average costs the addressee is faced with during a period long enough to be of climatological relevance are expressed by equation (5) in Section 5.2. Making use of some elementary algebra, one reads (with "F" meaning here "false alarm rate"):

$$\begin{aligned}
M_R &= \frac{1}{a+b+c+d}[bL + C\,c + (C+\lambda)d] \\
&= \frac{1}{a+b+c+d}[Cc + d(C+\lambda-L) + (b+d)L] \\
&= C\frac{c}{a+c}\cdot\frac{a+c}{a+b+c+d} + \frac{d}{b+d}\cdot\frac{b+d}{a+b+c+d}(C+\lambda-L) \\
&\quad + \frac{b+d}{a+b+c+d}\,L \\
&= C\,F\frac{a+b+c+d-b-d}{a+b+c+d} + H\Omega(C+\lambda-L) + \Omega\,L \\
&= C\,F\,(1-\Omega) + H\,\Omega(C+\lambda-L) + \Omega\,L \\
&= L\,[\Gamma\,F\,(1-\Omega) + H\,\Omega\,(\Gamma+\Lambda-1) + \Omega]
\end{aligned}$$

Finally:

$$M_{R(Hr,Far)} = L\,[\Gamma\,Far\,(1-\Omega) + Hr\,\Omega\,(\Gamma+\Lambda-1) + \Omega]$$

12.5.2 Derivation of $M_{A(Hr,Fao)}$

The average costs the addressee is faced with during a period long enough to be of climatological relevance are expressed by equations (5) or (8) in Sections 5.2 and 7.1. Making use of the substitution *ad infinitum* of $c = F(c+d)$ into itself (with "F" meaning now "false alarm ratio"), one reads:

$$
\begin{aligned}
M_A &= \frac{1}{a+b+c+d}[bL + Cc + (C+\lambda)d] \\
&= \frac{\Omega}{b+d}[bL + d\lambda + C(c+d)] \\
&= \Omega[\frac{b+d-d}{b+d}L + \frac{d}{b+d}\lambda + C\frac{c+d}{b+d}] \\
&= \Omega[(1-H)L + H\lambda + C\frac{F(c+d)+d}{b+d}] \\
&= \Omega[(1-H)L + H\lambda + C(H + F\frac{c+d}{b+d})] \\
&= \Omega[(1-H)L + H\lambda + C(H + F\frac{F(c+d)+d}{b+d})] \\
&= \Omega[(1-H)L + H\lambda + C(H + HF + F^2\frac{c+d}{b+d})] \\
&= \Omega[(1-H)L + H\lambda + C(H + HF + HF^2 + F^3\frac{c+d}{b+d})] \\
&= \cdots \\
&= \lim_{n\to\infty} \Omega[(1-H)L + H\lambda + C(H + HF + HF^2 + HF^3 + \cdots + F^n\frac{c+d}{b+d})] \\
&= \Omega[(1-H)L + H\lambda + \frac{CH}{1-F}]
\end{aligned}
$$

The following sequence (with $\eta = \frac{c+d}{b+d}$) has been considered :

$$S_n = H + H\ F + H\ F^2 + H\ F^3 + \cdots + H\ F^{n-1} + \eta\ F^n$$

It satisfies:

$$S_n\ (1-F) = H + (\eta - H)\ F^n - \eta\ F^{n+1}$$

and, therefore:

$$
\begin{aligned}
\lim_{n\to\infty} S_n &= \lim_{n\to\infty} \frac{1}{1-F}(H + (\eta - H)\ F^n - \eta\ F^{n+1}) \\
&= \frac{H}{1-F}
\end{aligned}
$$

That kind of computation delighted the Bernoulli brothers. Of course, the convergence of the infinite sequence occurs only for $F < 1$, a requirement which can be expected to hold for the false alarm ratio. Indeed, this condition will be anchored in the setting of the application of M_A. The expression can be rearranged in a scalar product as follow:

$$\begin{aligned}
M_{A(Hr,Fao)} &= \Omega[(1-Hr)\,L + Hr\,\lambda + \frac{C\,Hr}{1-Fao}] \\
&= \Omega L\,[(1-Hr) + Hr\,\frac{\lambda}{L} + \frac{C}{L}\,\frac{Hr}{1-Fao}] \\
&= \Omega L\,[(1-Hr) + Hr\,\Lambda + \Gamma\,\frac{Hr}{1-Fao}] \\
&= \Omega L\,\cdot \begin{bmatrix} 1 & \Lambda & \Gamma \end{bmatrix} \centerdot \begin{bmatrix} 1-Hr \\ Hr \\ Hr\,(1-Fao)^{-1} \end{bmatrix}
\end{aligned}$$

Once again do the climatic burden, the cost-loss and residual-loss ratios Γ and Λ nicely emerge. Together with the scalar product structure, they allow a cogent interpretation of the latter expression, presented in Section 7.1.

12.6 Determination of the optimal probability threshold P^*

As in Section 12.3, the derivation is carried through as a optimisation problem. However, the relationship between addressee's and issuer's profiles is now considered. The addressee's profile at meteorological threshold Q^* represents the utility function to be minimised under the constraint provided by the issuer's profile, expressed as his relative operation characteristic. Both sides of the following expression have therefore to be evaluated:

$$\frac{\partial_P Hr}{\partial_P Fao}|_{Issuer_{(Q^*)}} = \frac{\partial Hr}{\partial Fao}|_{Addressee_{(Q^*)}}$$

Issuer's profile For a given meteorological threshold Q^*, hit rates and false alarm ratios are extracted from the data emanating from the simulation for a sequence of probability thresholds spanning from $P = 0.05$ up to $P = 0.95$ in steps of 0.05. Polynomial fitting of that information delivers the values of the coefficients α_i and β_i of the polynomial expressions (of degree $N = 4$) given below.

$$Hr_{(p)} = \sum_{i=0}^{N} \alpha_i p^i \; ; \; Fao_{(p)} = \sum_{i=0}^{N} \beta_i p^i$$

The smoothed orange ROC-curves presented throughout the document have been elaborated with these polynomials for $p \in [0.05, 0.95]$. Such computations have to be performed for each value of the meteorological threshold.
Finally, the determination of the required quotient is obvious:

$$\frac{\partial_P Hr}{\partial_P Fao}|_{Issuer_{(Q^*)}} = (\sum_{i=1}^{N} i\alpha_i p^{i-1})(\sum_{i=1}^{N} i\beta_i p^{i-1})^{-1}$$

Of course, some numerical checks have to be implemented in the simulations in order the prevent their blow-up in case of zero denominator.

Addressee's profile Considering that the derivation is to be realised for false alarm ratios lying below the limit induced by the climatic burden (main text, Section 7.1), the expression for $M_{A(Hr,Fao)}$ given at the end of Section 12.5 is preferred to the full-fledged formulation of $\mathcal{M}_{Q^*(Hr,Fao),\mathcal{S}}$. Taking into account the continuity of $M_{A(Hr,Fao)}$ with respect to its variables Hr and Fao, the theorem of the implicit function is applied in the sense that the iso-costs of $M_{A(Hr,Fao)}$ are simply its implicit functions. On any iso-cost of monetary value $M_\$$ the relation $M_{A(Hr,Fao)} = M_\$$ holds for the corresponding Hr and Fao values. Thus, if Hr is the implicit function expressed in terms of Fao, the total derivative is:

$$D_{Fao} M_A(Hr_{(Fao)}, Fao) = \frac{\partial M_A}{\partial Hr}\frac{\partial Hr}{\partial Fao} + \frac{\partial M_A}{\partial Fao} = 0.$$

After few algebraic manipulations, the result reads:

$$\begin{aligned}\frac{\partial Hr}{\partial Fao}|_{Addressee} &= -\frac{\partial M_A}{\partial Fao}(\frac{\partial M_A}{\partial Hr})^{-1} \\ &= -\frac{\Gamma\, Hr}{1 - Fao} \cdot \frac{1}{\Gamma + (1 - Fao)(\Lambda - 1)} \\ &= \frac{E_{(Q^*)}\, Hr}{(Fao - 1)(E_{(Q^*)} + Fao - 1)}\end{aligned}$$

According to the definition of a rational addressee, the cost-loss ratio having been substituted by its definition $\Gamma_{(Q^*)} = (1 - \Lambda)\, E_{(Q^*)}$ in the last expression, the Λ ratio vanishes as well, leaving only the value of the disaster profile $E_{(Q^*)}$ in the equation.

Synthesis Requiring the identity of the slopes at the point on the ROC with curvilinear abscissa P and accordingly equating the two quantities:

$$\frac{\partial_P Hr}{\partial_P Fao}|_{Issuer_{(Q^*)}} = \frac{\partial Hr}{\partial Fao}|_{Addressee_{(Q^*)}}$$

leads to:

$$(\sum_{i=1}^{N} i\alpha_i p^{i-1})(\sum_{i=1}^{N} i\beta_i p^{i-1})^{-1} = \frac{E_{(Q^*)}\, Hr}{(Fao-1)(E_{(Q^*)}+Fao-1)}$$

substituing Hr and Fao by their polynomial expressions on the right hand side:

$$(\sum_{i=1}^{N} i\alpha_i p^{i-1})(\sum_{i=1}^{N} i\beta_i p^{i-1})^{-1} = \frac{E_{(Q^*)}\, \sum_{i=0}^{N} \alpha_i p^i}{(\sum_{i=0}^{N} \beta_i p^i - 1)(E_{(Q^*)} + \sum_{i=0}^{N} \beta_i p^i - 1)}$$

and rearranging provides the following polynomial equation $\mathcal{P}_{Q^*(p)} = 0$ in unknown p (of degree 11 for $N = 4$):

$$\begin{aligned}
\mathcal{P}_{Q^*(p)} &= \sum_{i=1}^{N} i\alpha_i p^{i-1}(\sum_{i=0}^{N} \beta_i p^i - 1)(E_{(Q^*)} + \sum_{i=0}^{N} \beta_i p^i - 1) \\
&- E_{(Q^*)} \sum_{i=1}^{N} i\beta_i p^{i-1} \sum_{i=0}^{N} \alpha_i p^i \\
&= 0
\end{aligned}$$

whose only real root is the probability threshold P^* corresponding to the meteorological threshold Q^*. The location of the green dots pictured in all figures from Figure 15 onward, computed from the solution P^* is given by the co-ordinates $\{Hr_{(P^*)},\ Fao_{(P^*)}\}$.

This polynomial equation directly connects the forecasting skill of the issuer to the disaster profile of the addressee. All economic parameters, i.e. costs, losses, residual losses, as well as the deduced parameters Γ and Λ, after having served as scaffoldings in the elaboration of the relationship between both actors, disappear in this definitive formulation.

Remarks The differential equation derivated for the issuer's profile can be integrated for itself, yielding to the expression of the iso-costs in the

$\{Hr, Fao\}$ frame. This expression, not given here, has been used to draw the green addressee's profiles in all figures from Figure 15 onward. Furthemore, assuming that an numerical estimation $\frac{\Delta Hr}{\Delta Fao}$ of the differential ratio $\frac{\partial Hr}{\partial Fao}$ is known, an interesting relationship between both actor's profiles can be derivated. The disaster profile $E_{(Q)}$ is isolated first:

$$E_{(Q)} = \frac{(Fao-1)^2}{Hr\frac{\Delta Fao}{\Delta Hr} - Fao + 1}$$

Then, using the definitions of Γ and Λ, and equation (7), one derives:

$$\frac{C}{L-\lambda} = \frac{\Gamma}{1-\Lambda} = E_{(Q)} = \frac{(Fao-1)^2}{Hr\frac{\Delta Fao}{\Delta Hr} - Fao + 1}$$

This expression is discussed in Section 7.3.

12.7 Conceiving the efficiency measure of a warning system

The efficiency measure proposed is based upon two assumptions.

The first one is that the addressee chooses either to assume his climatic burden ΩL, thus never takes mitigating or protective actions, or he cooperates with a warning issuer. In this constellation, no contract being considered between the addressee and an insurance company, the parameter setting $\mathcal{S} = 1$ given in the definition of equation (11) for $\mathcal{M}_{Q(Hr,Fao),\mathcal{S}}$ remains valid. This first assumption provides us with the upper bound - ΩL - of the efficiency measure.

The second assumption, defining the bottom bound of the efficiency measure, identifies that bound with the performance provided by an hypothetical perfect warning system whose hit rate would be one and false alarm ratio zero. Accordingly, such a bottom bound would given by $\mathcal{M}_{Q(1,0),\mathcal{S}}$.

Then, in this set-up, the efficiency is given by the difference between the climatic burden ΩL and the burden occurring when warnings are issued and mitigating actions taken at a given meteorological threshold, hit rate and false alarm ratio. This difference is given by $\Omega L - \mathcal{M}_{Q(Hr,Fao),\mathcal{S}}$ (> 0).

Considering that $\mathcal{M}_{Q(Hr,Fao),\mathcal{S}} \in [\mathcal{M}_{Q(1,0),\mathcal{S}},\ \Omega L]$, the efficiency measure is normalized between zero and one, and reads:

$$\mathcal{F}_{Q(Hr,Fao),\mathcal{S}} = \frac{\Omega L - \mathcal{M}_{Q(Hr,Fao),\mathcal{S}}}{\Omega L - \mathcal{M}_{Q(1,0),\mathcal{S}}}$$

Taking finally into account the definition (11) of $\mathcal{M}_{Q(Hr,Fao)}$ and the aforementioned definition of $\mathcal{F}_{Q(Hr,Fao)}$, one notices that the previous expression can be simplified by the climatic burden ΩL, thus giving:

$$\mathcal{F}_{Q(Hr,Fao),\mathcal{S}} = \frac{1 - Min_{Q,(Hr,Fao),\mathcal{S}}}{1 - Min_{Q,(1,0),\mathcal{S}}}$$

The efficiency is zero when a warning system brings no positive departure in the addressee's financial burden from his climatic burden. It is one when the warning system, working perfectly, detects all events and issues no false alarms.

It is worth noticing that, following the observation already made in Section 10.5 related to the disappearance of the financial parameters in the differential formulation of the addressee's profile, the climatic burden ΩL vanishes here as well. Were, however, an insurance company entering the game, its role could be taken into account by the adequate tuning of the $\mathcal{S}$ parameter.

12.8 Assessing the scheme for variable Disaster Costs

The verification is performed throughout the above Sections of the Appendix, and partially in the main text.

Section 12.1 is not concerned.

Section 12.2. The problem is connected to the Section 5.1.1 in the main text. $\mathcal{E}_{m(Q)} = E(Q)\, m(Q)$, as defined in Section 10.1, is substituted to $E_{(Q)}$ in expression $E_{(q)}dq = \Pr[\text{Event occurs during } \Delta \text{ by weather in } [q, q+dq]]\ ;\ q \in [0, B]$. Considering the argument of equivalence and the introduction of the corresponding factor $m_{(Q)}$ proposed in the main text, Section 10.1, this substitution is sustainable under the requirement that $\mathcal{E}_{m(Q)} < 1\ \forall Q \in [0, B]$. Then the structure of the table in Section 12.2 remains unchanged. The substitution $\mathcal{E}_{m(Q)} = E(Q)\, m(Q) \longmapsto E_{(Q)}$ occurs here too, as well as in the definition of $\Omega \longmapsto \displaystyle\int_0^B C(q)\ \mathcal{E}_{m(Q)} dq$. Under such conditions, the computations of the probabilistic hit rate and false alarm rate remain consistent.

Section 12.3 and Section 5.2: The equation (5) has to be considered first: $M = \frac{1}{a+b+c+d}\,[bL + Cc + (C+\lambda)d]$. The argument of equivalence and the introduction of the corresponding factor $m_{(Q)}$ proposed in the main text, Section 9.2, operate on the b factor of equation (5) which is transformed as $b \longmapsto b\ m_{(Q)}$ in numerator as well as in denominator of equation (5). Both are therefore taken into account in the computation of the hit rate and the false alarm rate, as well as for the estimation of the Ω factor in the subsequent computation. In Section 12.3 the derivations of the risk profile and the economic profile are coherently performed whit the aforementioned substitutions and under the requirement $\mathcal{E}_{m(Q)} < 1\ \forall Q \in [0, B]$. Finally, the definition of the cost-loss ratio reads: $\Gamma_{(Q)} = \dfrac{C}{L} = (1-\Lambda)\,\mathcal{E}_{m(Q)}$.

Section 12.4 is not concerned.

Section 12.5 and Section 7.1: Equation (8) is identical to equation (5). The argument presented in Sections 12.3 and 5.2 therefore holds with the false alarm rate replaced by the false alarm ratio. Furthermore, in equation (11) the climatic burden ΩL is extracted in the computation of the minimum.

Section 12.6: The derivation of the addressee's profile occurs as previously with substitution $\mathcal{E}_{m(Q)} = E(Q)\,m(Q) \longmapsto E_{(Q)}$. The climatic burden ΩL was extracted in the definition of equation (11).

Section 12.7: The efficiency measure being defined as a ratio, equation (13), it is independent from the climatic burden.

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